46. $\begin{vmatrix} 0 & xyz & x-z \\ y-x & 0 & y-z \\ z-x & z-y & 0 \end{vmatrix} = \ldots\ldots\ldots\ldots$

Ans: Here, we have $\begin{vmatrix} 0 & xyz & x-z \\ y-x & 0 & y-z \\ z-x & z-y & 0 \end{vmatrix}$

Applying $C_1 \to C_1 - C_3$, we get

$$= \begin{vmatrix} z-x & xyz & x-z \\ z-x & 0 & y-z \\ z-x & z-y & 0 \end{vmatrix}$$

Taking common $(z-x)$ from C_1

$$= (z-x) \begin{vmatrix} 1 & xyz & x-z \\ 1 & 0 & y-z \\ 1 & z-y & 0 \end{vmatrix}$$

Applying $[R_2 \to R_2 - R_1 \ and \ \ R_3 \to R_3 - R_1]$, we get

$$= (z-x) \begin{vmatrix} 1 & xyz & x-z \\ 0 & -xyz & y-x \\ 0 & z-y-xyz & z-x \end{vmatrix}$$

Expanding along C_1

$$= (z-x)\,[-xyz\,(z-x) - (y-x)\,(z-y-xyz)]$$

$$= (z-x)\left[-xyz^2 + x^2yz - \left(yz - y^2 - xy^2z - xz + xy + x^2yz\right)\right]$$

$$= (z-x)\left[-xyz^2 - \ yz + y^2 + xy^2z + xz - xy\right]$$

$$= (z-x)\left[y^2 - \ yz + xy^2z - xyz^2 + xz - xy\right]$$

$$= (z-x)\,[y\,(y-z) + xyz\,(y-z) - x\,(y-z)]$$

$$= (z-x)\,(y-z)\,(y+xyz-x)$$

Hence, $\begin{vmatrix} 0 & xyz & x-z \\ y-x & 0 & y-z \\ z-x & z-y & 0 \end{vmatrix} = (z-x)\,(y-z)\,(y+xyz-x)\,.$