Related Questions with Solutions

Questions

Quetion: 01

If A, B, C are angles of a triangle and $\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix} = 0$, then

A. $\triangle ABC$ must be isosceles.
B. $\triangle ABC$ must be right angled
C. $\triangle ABC$ must be equilateral
D. $\triangle ABC$ must be scalene

Solutions

Solution: 01

Let

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1+\sin A & \sin B-\sin A & \sin C-\sin A \\ \sin A+\sin^2 A & (\sin B-\sin A)(\sin B+\sin A+1) & (\sin C-\sin A)(\sin C+\sin A+1) \end{vmatrix}$$

$[C_2 \to C_2 - C_1, C_3 \to C_3 - C_1]$

$= (\sin B-\sin A)(\sin C-\sin A)(\sin C-\sin B)$

Now, since Δ is given to be zero, therefore we have

$(\sin B-\sin A)(\sin C-\sin A)(\sin C-\sin B) = 0$

i.e. $\sin B-\sin A = 0$

or $\sin C-\sin A = 0$ or $\sin C-\sin B = 0$

i.e. $\sin B = \sin A$ or $\sin C = \sin A$ or $\sin C = \sin B$

i.e. B = A

or C = A

or C = B

In all the three cases, the triangle will be isosceles.

Correct Options

Answer:01

Correct Options: A