Related Questions with Solutions

Questions

Quetion: 01

If $A = \begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$, then A is

A. invertible only if $t = \pi$
B. invertible only if $t = \pi/2$
C. not invertible for any $t \in R$
D. invertible for all $t \in R$

Solutions

Solution: 01

Given,

$$A = \begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$$

Now, $|A| = e^{-t}\begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t - \sin t & -\sin t + \cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$

$= e^{-t}\begin{vmatrix} 1 & \cos t & \sin t \\ 0 & -\sin t - 2\cos t & \cos t - 2\sin t \\ 0 & 2\sin t - \cos t & -2\cos t - \sin t \end{vmatrix}$ [Applying $R2 \blacklozenge R2 - R1$ and $R3 \blacklozenge R3 - R1$]

$= e^{-t}[(-\sin t - 2\cos t)(-2\cos t - \sin t) - (\cos t - 2\sin t)(2\sin t - \cos t)]$

$= e^{-t}\left[5\cos^2 t + 5\sin^2 t\right] = 5e^{-t} \neq 0 \quad \forall t \in R$

$\therefore \quad A$ is invertible $\forall t \in R$.

Correct Options

Answer:01
Correct Options: D