

## Matrices and Determinants - Class XII

### Past Year JEE Questions

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#### Questions

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##### Question: 01

Let  $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$  and  $Q = [q_{ij}]$  be two  $3 \times 3$  matrices such that  $Q - P^5 = I_3$ .

Then  $\frac{q_{22} + q_{31}}{q_{32}}$  is equal to :

- A. 15
- B. 9
- C. 135
- D. 10

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#### Solutions

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##### Solution: 01

##### Explanation

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 3+3 & 1 & 0 \\ 9+9+9 & 3+3 & 1 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 1 & 0 & 0 \\ 3+3+3 & 1 & 0 \\ 6 \cdot 9 & 3+3+3 & 1 \end{bmatrix}$$

$$P^n = \begin{bmatrix} 1 & 0 & 0 \\ 3n & 1 & 0 \\ \frac{n(n+1)}{2} \cdot 9 & 3n & 1 \end{bmatrix}$$

$$P^5 = \begin{bmatrix} 1 & 0 & 0 \\ 5 \cdot 3 & 1 & 0 \\ 15 \cdot 9 & 5 \cdot 3 & 1 \end{bmatrix}$$

$$Q = P^5 + I_3$$

$$Q = \begin{bmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{bmatrix}$$

$$\frac{q_{22} + q_{31}}{q_{32}} = \frac{15 + 135}{15} = 10$$

Aliter

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{pmatrix}$$

$$P = I + X$$

$$X = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{pmatrix}$$

$$X^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 0 & 0 \end{pmatrix}$$

$$X^3 = 0$$

$$P^5 = I + 5X + 10X^2$$

$$Q = P^5 + I = 2I + 5X + 10X^2$$

$$Q = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 15 & 0 & 0 \\ 15 & 15 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 90 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow Q = \begin{pmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{pmatrix}$$