

## Matrices and Determinants - Class XII

### Past Year JEE Questions

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#### Questions

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##### Question: 01

Let  $A$  be a matrix such that  $A \cdot \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$  is a scalar matrix and  $|3A| = 108$ .

Then  $A^2$  equals :

A.  $\begin{bmatrix} 4 & -32 \\ 0 & 36 \end{bmatrix}$

B.  $\begin{bmatrix} 36 & 0 \\ -32 & 4 \end{bmatrix}$

C.  $\begin{bmatrix} 4 & 0 \\ -32 & 36 \end{bmatrix}$

D.  $\begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$

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#### Solutions

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##### Solution: 01

##### Explanation

According to questions,

$$A \cdot \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}^{-1}$$

$$\Rightarrow A = \frac{1}{3} \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} 1 & -\frac{2}{3} \\ 0 & \frac{1}{3} \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} \lambda & -\frac{2}{3}\lambda \\ 0 & \frac{\lambda}{3} \end{bmatrix}$$

As  $|3A| = 108$

$$\Rightarrow 108 = \begin{vmatrix} 3\lambda & -2\lambda \\ 0 & \lambda \end{vmatrix}$$

$$\Rightarrow 3\lambda^2 = 108$$

$$\Rightarrow \lambda^2 = 36$$

$$\Rightarrow \lambda = \pm 6$$

When  $\lambda = +6$

$$\text{then } A = \begin{bmatrix} 6 & -4 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$$

For  $\lambda = -6$

$$A = \begin{bmatrix} -6 & 4 \\ 0 & -2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$$