58. The maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$ **is** $\frac{1}{2}$.

Ans: Here, we have $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+sin\theta & 1 \\ 1 & 1 & 1+cos\theta \end{vmatrix}$

Applying $[C_1 \to C_1 - C_3]$ and $[C_2 \to C_2 - C_3]$, we get

$$\Rightarrow \Delta = \begin{vmatrix} 0 & 0 & 1 \\ 0 & sin\theta & 1 \\ -cos\theta & -cos\theta & 1 \end{vmatrix}$$

Now, expanding along R_1

$$\Rightarrow \Delta = 1\,(0 + sin\theta.\,cos\theta)$$

$$\Rightarrow \Delta = \sin\theta.\cos\theta$$

$$\Rightarrow \Delta = \frac{2}{2}sin\theta.\,cos\theta$$

$$\Rightarrow \Delta = \frac{1}{2}sin2\theta$$

Also, we know that, \[- {{1}} \leqslant \sin {{2\theta }} \leqslant {{1}}\]

$$\Rightarrow -\frac{1}{2} \leqslant \frac{1}{2}\sin 2\theta \leqslant \frac{1}{2}$$

$$\Rightarrow -\frac{1}{2} \leqslant \Delta \leqslant \frac{1}{2}$$

Thus, maximum value of given determinant is $\frac{1}{2}$

Hence, the given statement is true.