53. $\begin{vmatrix} \mathbf{x+1} & \mathbf{x+2} & \mathbf{x+a} \\ \mathbf{x+2} & \mathbf{x+3} & \mathbf{x+b} \\ \mathbf{x+3} & \mathbf{x+4} & \mathbf{x+c} \end{vmatrix} = 0$, where $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are in $\mathbf{A.P.}$

Ans: Here, we have $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$

Applying $C_1 \rightarrow C_1 - C_2$, we get

$= \begin{vmatrix} -1 & x+2 & x+a \\ -1 & x+3 & x+b \\ -1 & x+4 & x+c \end{vmatrix}$

Applying

$$R_2 \rightarrow R_2\, R_1 \text{ and } R_3 \rightarrow R_3\, R_1$$

, we get

$= \begin{vmatrix} -1 & x+2 & x+a \\ 0 & 1 & b-a \\ 0 & 2 & c-a \end{vmatrix}$

Expanding along C_1

$= -1\,[(c-a) - 2\,(b-a)]$

$= -1\,(c-a-2b+2a]$

$= -1\,(c-2b+a]$

Since, a, b and c are in A.P.