Matrices and Determinants - Class XII

Related Questions with Solutions

Questions

Quetion: 01

For two 3×3 matrices A and B, let $A + B = 2B'$ and $3A + 2B = I_3$, where B' is the transpose of B and I_3 is 3 × 3 identity matrix. Then

A. $10A + 5B = 3I_3$
B. $5A + 10B = 2I_3$
C. $3A + 6B = 2I_3$
D. $B + 2A = I_3$

Solutions

Solution: 01

Given, $A + B = 2B'$

$\Rightarrow (A+B)' = (2B')' \Rightarrow A' + B' = 2B \Rightarrow B = \dfrac{A' + B'}{2}$

Now, $A + \left(\dfrac{B' + A'}{2}\right) = 2B' \quad [\because A + B = 2B']$

$\Rightarrow \quad 2A + B' + A' = 4B' \Rightarrow 2A + A' = 3B'$

$\Rightarrow \quad A = \dfrac{3B' - A'}{2}$

Also, $3A + 2B = I_3$ ...[1]

$\Rightarrow 3\left(\dfrac{3B' - A'}{2}\right) + 2\left(\dfrac{A' + B'}{2}\right) = I_3$

$\Rightarrow \left(\dfrac{9B' + 2B'}{2}\right) + \left(\dfrac{2A' - 3A'}{2}\right) = I_3$

$\Rightarrow \quad 11B' - A' = 2I_3 \Rightarrow (11B' - A')' = (2I_3)'$

$\Rightarrow \quad 11B - A = 2I_3$ ...[2]

Multiplying [2] by 3 and then adding [1] and [2], we get

$35B = 7I_3 \Rightarrow B = \dfrac{I_3}{5}$

From (2), $11\dfrac{I_3}{5} - A = 2I_3 \Rightarrow 11\dfrac{I_3}{5} - 2I_3 = A$

$\Rightarrow A = \dfrac{I_3}{5}$

$\therefore \quad 5A = 5B = I_3 \Rightarrow 10A + 5B = 3I_3$

Correct Options

Answer:01
Correct Options: A