Matrices and Determinants - Class XII

Past Year JEE Questions

Questions

Quetion: 01

Let α and β be the roots of the equation $x^2 + x + 1 = 0$. Then for $y \neq 0$ in R,

$$\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$$

is equal to

A. $y(y^2 - 1)$

B. $y(y^2 - 3)$

C. y^3

D. $y^3 - 1$

Solutions

Solution: 01

Explanation

α and β are the roots of the equation $x^2 + x + 1 = 0$.

$\therefore \alpha = \omega$ and $\beta = \omega^2$

$$\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$$

$$= \begin{vmatrix} y+1 & \omega & \omega^2 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

$C_1 \rightarrow C_1 + C_2 + C_3$

$$= \begin{vmatrix} y+1+\omega+\omega^2 & \omega & \omega^2 \\ y+1+\omega+\omega^2 & y+\omega^2 & 1 \\ y+1+\omega+\omega^2 & 1 & y+\omega \end{vmatrix}$$

$$= \begin{vmatrix} y & \omega & \omega^2 \\ y & y+\omega^2 & 1 \\ y & 1 & y+\omega \end{vmatrix}$$

As $1 + \omega + \omega^2 = 0$

$$= y\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & y+\omega^2 & 1 \\ 1 & 1 & y+\omega \end{vmatrix}$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$$= y\begin{vmatrix} 1 & \omega & \omega^2 \\ 0 & y+\omega^2-\omega & 1-\omega^2 \\ 0 & 1-\omega & y+\omega-\omega^2 \end{vmatrix}$$

$= y\left[\left(y+\omega^2-\omega\right)\left(y+\omega-\omega^2\right)-\left(1-\omega^2\right)(1-\omega)\right]$

$= y(y^2) = y^3$