

Matrices and Determinants - Class XII

Past Year JEE Questions

Questions

Question: 01

The total number of matrices

$$A = \begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix}$$

($x, y \in \mathbb{R}, x \neq y$) for which $A^T A = 3I_3$ is :-

- A. 3
- B. 4
- C. 2
- D. 6

Solutions

Solution: 01

Explanation

Given $A^T A = 3I_3$

$$\Rightarrow \begin{bmatrix} 0 & 2x & 2x \\ 2y & y & -y \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{bmatrix}$$

$$= 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\therefore 8x^2 = 3, 6y^2 = 3$$

$$\Rightarrow x = \pm\sqrt{\frac{3}{8}}, y = \pm\sqrt{\frac{1}{2}}$$

Total possible combination of x and $y = 2 \times 2 = 4$