Exemplar Problem Matrix and Determinants

44. If $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$, and $A^{-1} = A'$, find value of a. Solution: $\begin{aligned} \mathbf{Given,} \\ A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \text{ and } A' = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \end{aligned}$ Also, $\begin{aligned} A^{-1} = A' \\ AA^{-1} = AA' \\ I = AA' \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & 0 \\ 0 & \sin^2 \alpha + \cos^2 \alpha \end{bmatrix}$

By using equality of matrices, we get

 $\cos^2 \alpha + \sin^2 \alpha$ = 1, which is true for all real values of α .