11. Show that satisfies the equation $A^2 - 3A - 7I = 0$ and hence find A^{-1}.

Solution:

Given,

$$A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$$

$$\textbf{So, } A^2 = A \cdot A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} \cdot \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 25-3 & 15-6 \\ -5+2 & -3+4 \end{bmatrix} = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix}$$

$$3A = 3\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix}$$

$$\textbf{And, } 7I = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\textbf{Hence, } A^2 - 3A - 7I = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 22-15-7 & 9-9-0 \\ -3+3-0 & 1+6-7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

Now,

$A^2 - 3A - 7I = 0$

Multiplying both sides with A^{-1}, we get

$A^{-1} [A^2 - 3A - 7I] = A^{-1} 0$

$A^{-1}. A . A - 3A^{-1}. A - 7A^{-1}. I = 0$

$I . A - 3I - 7A^{-1} = 0$ [As $A^{-1}. A = I$]

$A - 3I - 7A^{-1} = 0$

$7A^{-1} = A - 3I$

$$7A^{-1} = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$$

Therefore, $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$