In a bolt factory, machines A, B and C manufacture 25%, 35%, 40% respectively. Of the total of their output 5, 4 and 2% are defective. A bolt is drawn and is found to be defective. What are the probabilities that it was manufactured by the machines A, B and C?

A $\dfrac{25}{69}, \dfrac{28}{69}, \dfrac{16}{69}$.

B $\dfrac{25}{69}, \dfrac{27}{69}, \dfrac{17}{69}$.

C $\dfrac{28}{69}, \dfrac{25}{69}, \dfrac{16}{69}$.

D $\dfrac{27}{69}, \dfrac{26}{69}, \dfrac{16}{69}$.

Correct option is A)

Here $P(A) = \dfrac{25}{100}, P(B) = \dfrac{35}{100}, P(C) = \dfrac{40}{100}$

$P\left(\dfrac{D}{A}\right) = \dfrac{5}{100}, P\left(\dfrac{D}{B}\right) = \dfrac{4}{100}, P\left(\dfrac{D}{C}\right) = \dfrac{2}{100}$

where D denotes defective bolts

Now $P(D) = P(A).P\left(\dfrac{D}{A}\right) + P(B).P\left(\dfrac{D}{B}\right) + P(C).P\left(\dfrac{D}{C}\right)$

$= \dfrac{25}{100}.\dfrac{5}{100} + \dfrac{35}{100}.\dfrac{4}{100} + \dfrac{40}{100}.\dfrac{2}{100} = 0.0345$

$P\left(\dfrac{A}{D}\right) = \dfrac{P(A).P\left(\frac{D}{A}\right)}{P(A).P\left(\frac{D}{A}\right) + P(B).P\left(\frac{D}{B}\right) + P(C).P\left(\frac{D}{C}\right)}$

$= \dfrac{\frac{25}{100}.\frac{5}{100}}{0.0345} = \dfrac{25}{69}$

$P\left(\dfrac{B}{D}\right) = \dfrac{P(B).P\left(\frac{D}{B}\right)}{P(A).P\left(\frac{D}{A}\right) + P(B).P\left(\frac{D}{B}\right) + P(C).P\left(\frac{D}{C}\right)}$

$= \dfrac{\frac{35}{100}.\frac{4}{100}}{0.0345} = \dfrac{28}{69}$

$P\left(\dfrac{C}{D}\right) = \dfrac{P(C).P\left(\frac{D}{C}\right)}{P(A).P\left(\frac{D}{A}\right) + P(B).P\left(\frac{D}{B}\right) + P(C).P\left(\frac{D}{C}\right)}$

$= \dfrac{\frac{40}{100}.\frac{2}{100}}{0.0345} = \dfrac{16}{69}$