In a test an examinee either guesses or copies of knows the answer to a multiple choice question with four choices. The probability that he make a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that his answer is correct given that he copied it, is 1/8. find the probability that he knew the answer to the question given that he correctly answered it

Let E_1, E_2, E_3 and A be the events defined as

$E_1 \rightarrow$ The exaninee guesses the answer.

$E_2 \rightarrow$The examinee copies the answer.

$E_3 \rightarrow$ The examinee knows the answer.

and $A \rightarrow$ The examinee answer correctly.

We have,

$$P(E_1) = \frac{1}{3}, P(E_2) = \frac{1}{6}.$$

Since, E_1, E_2, E_3 are mutually exclusive and exhaustive events.

$$\therefore \quad P(E_1) + P(E_2) + P(E_3) = 1$$

$$\Rightarrow P(E_3) = 1 - \frac{1}{3} - \frac{1}{6} = \frac{1}{2}$$

If E_1 has already occured, then the examinee guesses. Since, there are four choices out of which only one is correct, therefore the probability that he answer correctly given that he has made a quess is 1/4 ie

$P(A/E_1) = 1/4$

It is given that $(P(A/E_2) = 1/8$ and $P(A/E_3)$ = probability that he answer correctly given that he know the answer = 1.

By Baye's theorem, we have

$$P(E_3/A) = \frac{P(E_3) \cdot P(A/E_3)}{\left[P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + P(E_3) \cdot P(A/E_3)\right]}$$

$$\therefore \quad P(E_3/A) = \frac{\frac{1}{2} \times 1}{\left(\frac{1}{3} \times \frac{1}{4}\right) + \left(\frac{1}{6} \times \frac{1}{8}\right) + \left(\frac{1}{2} \times 1\right)} = \frac{24}{29}$$