

For three events A, B and C, $P(\text{Exactly one of A or B occurs}) = P(\text{Exactly one of B or C occurs}) = P(\text{Exactly one of C or A occurs}) = \frac{1}{4}$ and $P(\text{All the three events occur simultaneously}) = \frac{1}{16}$. Then the probability that at least one of the events occurs, is.

A $\frac{7}{32}$

B $\frac{7}{16}$

C $\frac{7}{64}$

D $\frac{3}{16}$

Correct option is B)

$$P(\text{exactly one of A or B}) = P(A \cup B) - P(A \cap B) = \frac{1}{4} = P(A) + P(B) - 2P(A \cap B)$$

$$P(\text{exactly one of C or B}) = P(C \cup B) - P(C \cap B) = \frac{1}{4} = P(C) + P(B) - 2P(C \cap B)$$

$$P(\text{exactly one of A or C}) = P(A \cup C) - P(A \cap C) = \frac{1}{4} = P(A) + P(C) - 2P(A \cap C)$$

Adding all

$$2P(A) + 2P(B) + 2P(C) - 2P(A \cap B) - 2P(A \cap C) - 2P(B \cap C) = \frac{3}{4}$$

$$P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) = \frac{3}{8}$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$= \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$$