Concepts and Formulas

Three Dimensional Geometry

Coplanarity of Two Lines In 3D Geometry

Let 2 lines are

$$\frac{x-x1}{l1} = \frac{y-y1}{m1} = \frac{z-z1}{n1} \ \& \ \frac{x-x2}{l2} = \frac{y-y2}{m2} = \frac{z-z2}{n2}$$

Two lines are coplanar iff

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

Calculation of Angle Between Two plane in the Cartesian Plane

Let $A_1 x + B_1 y + C_1 z + D_1 = 0$ and $A_2 x + B_2 y + C_2 z + D_2 = 0$ be the equation of two planes aligned to each other at an angle θ where A_1, B_1, C_1 and A_2, B_2, C_2 are the direction ratios of the normal to the planes, then the cosine of the angle between the two planes is given by:

$$\cos\theta = \left| \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\sqrt{A_2^2 + B_2^2 + C_2^2}} \right|$$

Angle between a Line and a Plane

A line is inclined at Φ to a plane. The vector equation of the line is given by $\vec{r} = \vec{a} + \lambda \vec{b}$ and the vector equation of the plane can be given by

$\vec{r}.\hat{n}$
= d

Let θ be the angle between the line and the normal to the plane. Its value can be given by the following equation:

$$\cos\theta = \left|\frac{\vec{b}\,.\,\vec{n}}{|\vec{b}|.|\vec{n}|}\right|$$

Φ is the angle between the line and the plane which is the complement of θ or 90 – θ. We know that cos θ is equal to sin (90 – θ). So Φ can be given by:

sin (90 – θ) = cos θ

Important Notes on Distance Between Point and Plane

- Distance Between Point and Plane Formula: $|Ax_o + By_o + Cz_o + D|/\sqrt{(A^2 + B^2 + c^2)}$
- Distance Between Point and Plane is zero if the given point lies on the given plane.

The distance between two parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is given by

$$\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}.$$