25. Show that the points

$$\left(\hat{i} - \hat{j} + 3\widehat{k}\right)$$

and

$$3\left(\hat{i} + \hat{j} + \widehat{k}\right)$$

are equidistant from the plane

$$\overrightarrow{r}\left[5\hat{i} + 2\hat{j} - 7\widehat{k}\right] + 9 = 0$$

and lies on the opposite side of it.

Ans: To show that these given points $\left(\hat{i} - \hat{j} + 3\hat{k}\right)$ and $3\left(\hat{i} + \hat{j} + \hat{k}\right)$ are equidistant from the plane $\overrightarrow{r}.\left[5\hat{i} + 2\hat{j} - 7\hat{k}\right] + 9 = 0$. We first find out the midpoint of the points which is $\left(2\hat{i} + \hat{j} + 3\hat{k}\right)$.

On substituting $\overrightarrow{r}$ by the midpoint in plane, we get

$$LHS = \left(2\hat{i} + \hat{j} + 3\hat{k}\right) . \left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9$$

$$= 10 + 2 - 21 + 9 = 0 = RHS$$

Hence, the two points lie on opposite sides of the plane are equidistant from the plane.