Concepts and Formulas

Three Dimensional Geometry

Concept of Plane in 3 Dimensional Geometry

A first degree equation in x, y, z represents a plane in 3D

$ax + by + cz = 0, z^2 b^2 + c^2 \neq 0$
represents a plane.

Normal Form of a Plane

Let P be the length of the normal from the origin to the plane and

l, m, n
be the direction cosines of that normal. Then the equation of the plane is given by
$lx + my + nz = P.$

Intercept form

Let a plane cuts length

a, b, c
from the coordinate axis.

Then equation of the plane is

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$

Planes passing through 3 given points

Plane passing through

$(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3)$
is

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0$$

Angle between 2 planes

$a_1 x + b_1 y + c_1 z + d_1 = 0$
and
$a_2 x + b_2 y + c_2 z + d_2 = 0$
is given by

$\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

Two sides of a plane

Consider 2 points

$A(x_1, y_1, z_1)$
and
$B(x_2, y_2, z_2)$

lie on the same side or opposite sides of a plane
$ax + by + cz + d = 0$
, accordingly as
$ax_1 + by_1 + cz_1 + d$
and
$ax_2 + by_2 + cz_2 + d$
are of same sign or opposite sign.

Distance from a point to a plane

Distance of a point (x_1,y_1,z_1) from a plane.

Distance of

(x_1, y_1, z_1)
from
$ax + by + cz + d$
is
$\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$

Equation of the planes bisecting the angle between 2 planes

Let

$a_1x + b_1y + c_1z + d = 0$
and
$a_2x + b_2y + c_2z + d = 0$
be 2 planes. The equation of plane bisecting the angles between them is

$\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}} = \pm\frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}$

Position of origin

The origin lies in the acute or obtuse angle between

$a_1x + b_1y + c_1z + d_1 = 0$
and
$a_2x + b_2y + c_2z + d_2 = 0$
according as
$a_1a_2 + b_1b_2 + c_1c_2 < 0 \; or \; > 0$
provided
d_1
and
d_2
are both positive.

Two Intersecting plane

If

$U = 0$
and
$V = 0$
be 2 planes then the plane passing through the line of their intersection is
$U + \lambda V = 0\lambda$
to be determined from given condition.