24. Find the equation of the plane through the intersection of the planes $\vec{r} \cdot (\hat{i} + 3\hat{j}) - 6 = 0$ and $\vec{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0$, whose perpendicular distance from origin is unity.

Solution:

Given planes are

$$\vec{r}.(\hat{i} + 3\hat{j}) - 6 = 0 \Rightarrow x + 3y - 6 = 0 \quad ...(i)$$

and $$\vec{r}.(3\hat{i} - \hat{j} - 4\hat{k}) = 0 \Rightarrow 3x - y - 4z = 0 \quad ...(ii)$$

Equation of the plane passing through the line of intersection of plane (i) and (ii) is

$$(x + 3y - 6) + k(3x - y - 4z) = 0 \quad ...(iii)$$

$$(1 + 3k)x + (3 - k)y - 4kz - 6 = 0$$

Perpendicular distance from origin

$$\Rightarrow \left| \frac{-6}{\sqrt{(1+3k)^2 + (3-k)^2 + (-4k)^2}} \right| = 1$$

$$\frac{36}{1 + 9k^2 + 6k + 9 + k^2 - 6k + 16k^2} = 1 \text{ [Squaring both sides]}$$

$$\frac{36}{26k^2 + 10} = 1 \Rightarrow 26k^2 + 10 = 36$$

$$26k^2 = 26 \Rightarrow k^2 = 1 \quad \therefore k = \pm 1$$

Putting the value of k in eq. (iii) we get,

$$(x + 3y - 6) \pm (3x - y - 4z) = 0$$

$\Rightarrow x + 3y - 6 + 3x - y - 4z = 0$ and $x + 3y - 6 - 3x + y + 4z = 0$

$\Rightarrow 4x + 2y - 4z - 6 = 0$ and $-2x + 4y + 4z - 6 = 0$

Thus, the required equations of planes are;

4x + 2y − 4z − 6 = 0 and -2x − 4y + 4z − 6 = 0.