

## Exemplar Problem

### Three Dimensional Geometry

15. Two systems of rectangular axis have the same origin. If a plane cuts them at distances  $a, b, c$  and  $a', b', c'$ , respectively, from the origin, prove that

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2} .$$

**Solution:**

Let's take  $OX, OY, OZ$  and  $ox, oy, oz$  to be two rectangular systems.

And, the equations of two planes are

$$\frac{X}{a} + \frac{Y}{b} + \frac{Z}{c} = 1 \dots(i) \quad \text{and} \quad \frac{x}{a'} + \frac{y}{b'} + \frac{z}{c'} = 1 \dots(ii)$$

Length of perpendicular from origin to plane (i) is

$$= \frac{\left| \frac{0}{a} + \frac{0}{b} + \frac{0}{c} - 1 \right|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$$

Length of perpendicular from origin to plane (ii)

$$= \frac{\left| \frac{0}{a'} + \frac{0}{b'} + \frac{0}{c'} - 1 \right|}{\sqrt{\frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}}} = \frac{1}{\sqrt{\frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}}}$$

As per the condition of the question

$$\frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} = \frac{1}{\sqrt{\frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}}}$$

Thus, 
$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}$$

$$\frac{X}{a} + \frac{Y}{b} + \frac{Z}{c} = 1 \dots(i) \quad \text{and} \quad \frac{x}{a'} + \frac{y}{b'} + \frac{z}{c'} = 1 \quad \dots(ii)$$

Length of perpendicular from origin to plane (i) is

$$= \frac{\left| \frac{0}{a} + \frac{0}{b} + \frac{0}{c} - 1 \right|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$$

Length of perpendicular from origin to plane (ii)

$$= \frac{\left| \frac{0}{a'} + \frac{0}{b'} + \frac{0}{c'} - 1 \right|}{\sqrt{\frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}}} = \frac{1}{\sqrt{\frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}}}$$

As per the condition of the question

$$\frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} = \frac{1}{\sqrt{\frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}}}$$

Thus, 
$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}$$