If α, β, γ are the cube roots of p, $p < 0$, then for any x, y & z then

$$\frac{x\alpha + y\beta + z\gamma}{x\beta + y\gamma + z\alpha} =$$

A ω

B ω^3

C ω^2

D 1

If $x^3 = 1$

$1, w, w^2$ are the cube roots of unity.

$\Longrightarrow 1 + w + w^2 = 0$

where, $w = \frac{-1+i\sqrt{3}}{2}$; $w^2 = \frac{-1-i\sqrt{3}}{2}$

Let z be the cube root of p.

$\Longrightarrow z^3 = p$

$\Longrightarrow z = p^{\frac{1}{3}}(\text{cis}\,(0))^{\frac{1}{3}}$ ...**{De Moivre's Theorem}**

$= p^{\frac{1}{3}}\text{cis}\,\left(\frac{2k\Pi}{3}\right)$

where, k=0,1,2.

$\alpha = p^{\frac{1}{3}}$

$\beta = p^{\frac{1}{3}}\text{cis}\,\left(\frac{2\Pi}{3}\right) = p^{\frac{1}{3}}w$

$\gamma = p^{\frac{1}{3}}\text{cis}\,\left(\frac{4\Pi}{3}\right) = p^{\frac{1}{3}}w^2$

$$\frac{x\alpha + y\beta + z\gamma}{x\beta + y\gamma + z\alpha} = \frac{x + yw + zw^2}{xw + yw^2 + z} = \frac{w^2\left(x + yw + zw^2\right)}{xw^3 + yw^4 + zw^2} = w^2$$