if $x = a + b$, $y = \alpha a + \beta b$ and $z = a\beta + b\alpha$, where $\alpha$ and $\beta$ are complex cube-roots of unity, then $xyz =$

A $a + b$

B $a^2 + b^2$

C $a^3 + b^3$

D $0$

$x = a + b$

$y = \alpha a + \beta b$

$z = \beta a + \alpha b$

Now, $\alpha$ and $\beta$ are complex cube roots of unity.

So,

$\alpha = \omega$

$\beta = \omega^2$

$$xyz = (a + b)\left(\omega a + \omega^2 b\right)\left(\omega^2 a + \omega b\right)$$

$$= (a + b)\left\{\omega^3 a^2 + \omega^2 ab + \omega^4 ab + \omega^3 b^2\right\}$$

$$= (a + b)\left\{\omega^3 a^2 + \omega^2 ab + \omega^1 ab + \omega^3 b^2\right\}$$

$$= (a + b)\left\{a^2 - ab + b^2\right\}$$

$$= a^3 + b^3$$