

If $a = \cos\left(\frac{2\pi}{7}\right) + i \sin\left(\frac{2\pi}{7}\right)$, then the quadratic equation whose roots are $\alpha = a + a^2 + a^4$ and $\beta = a^3 + a^5 + a^6$ is

A $x^2 - x + 2 = 0$

B $x^2 + x - 2 = 0$

C $x^2 - x - 2 = 0$

D $x^2 + x + 2 = 0$

$$a = \cos (2\pi / 7) + i \sin (2\pi / 7)$$

$$a^7 = [\cos (2\pi / 7) + i \sin (2\pi / 7)]^7$$

$$= \cos 2\pi + i \sin 2\pi = 1 \dots\dots(i)$$

$$S = \alpha + \beta = (a + a^2 + a^4) + (a^3 + a^5 + a^6)$$

$$S = a + a^2 + a^3 + a^4 + a^5 + a^6$$

=

$$\frac{a(1-a^6)}{1-a}$$

or S =

$$\frac{a-1}{1-a}$$

$$= -1 \dots\dots(ii)$$

$$P = \alpha * \beta = (a + a^2 + a^4) (a^3 + a^5 + a^6)$$

$$= a^4 + a^6 + a^7 + a^5 + a^7 + a^8 + a^7 + a^9 + a^{10}$$

$$= a^4 + a^6 + 1 + a^5 + 1 + a + 1 + a^2 + a^3 \text{ (From eqn (i))}$$

$$= 3 + (a + a^2 + a^3 + a^4 + a^5 + a^6)$$

$$= 3 + S = 3 - 1 = 2 \text{ [From (ii)]}$$

Required equation is, $x^2 - Sx + P = 0$

$$x^2 + x + 2 = 0.$$