Concepts and Formulas

Derivatives

Product Rule for Different Functions

Product Rule for Derivatives:

For any two functions, say f(x) and g(x), the product rule is D [f(x) g(x)] = f(x) D[g(x)] + g(x) D[f(x)]

$$d(uv)/dx = u(dv/dx)+ v(du/dx)$$

where u and v are two functions

Triple Product Rule:

Triple product rule is a generalization of product rule. If f(x), g(x) and h(x) be three differentiable functions, then the product rule of differentiation can be applied for these three functions as:

$$D[f(x). g(x). h(x)] = \{g(x). h(x)\} * D[f(x)] + \{f(x). h(x)\} * D[g(x)] + \{f(x). g(x)\} * D[h(x)]$$

Quotient Rule Formula

We can calculate the derivative or evaluate the differentiation of the quotient of two functions using the quotient rule derivative formula. The quotient rule derivative formula is given as,

$$f'(x) = [u(x)/v(x)]' = [v(x) \times u'(x) - u(x) \times v'(x)]/[v(x)]^2$$

where,

- f(x) = The function of the form u(x)/v(x) for which the derivative is to be calculated.
- u(x) = A differentiable function that makes numerator of the function f(x).
- u'(x) = Derivative of function u(x).
- v(x) = A differentiable function that makes denominator of the given function f(x).
- v'(x) = Derivative of the function v(x).

- There are connections between continuity and differentiability.
- Theorem: (Differentiability Implies Continuity) If **f** is a differentiable function at x = a, then f is continuous at x=a.

Important Formulas:

DERIVATIVE OF STANDARD FUNCTIONS

f(x)	$\frac{d}{dx}(f(x))$	f(x)	$\frac{d}{dx}(f(x))$
x^n	nx^{n-1}; $n \in R$	sec x	$\sec x \tan x$, $x \neq (2n+1)\frac{\pi}{2}$
e^x	e^x	cosec x	$-\text{cosec } x \cot x$; $x \neq n\pi$
x^x	$x^x(1 + \ln x)$	cot x	$-\text{cosec}^2 x$, $x \neq n\pi$
a^x	$a^x \log_e a$; $a > 0$, $a \neq 1$	$\sin^{-1}x$	$\frac{1}{\sqrt{1-x^2}}$; $-1 < x < 1$
$\log_e x$	$\frac{1}{x}$; $x > 0$	$\cos^{-1}x$	$-\frac{1}{\sqrt{1-x^2}}$; $-1 < x < 1$
$\log_a x$	$\frac{1}{x \log_e a}$; $x > 0$	$\tan^{-1}x$	$\frac{1}{1+x^2}$; $x \in R$
sin x	cos x	$\sec^{-1}x$	$\frac{1}{\lvert x \rvert \sqrt{x^2-1}}$; $\lvert x \rvert > 1$
cos x	$-\sin x$	$\text{cosec}^{-1}x$	$\frac{-1}{\lvert x \rvert \sqrt{x^2-1}}$; $\lvert x \rvert > 1$
tan x	$\sec^2 x$; $x \neq (2n+1)\frac{\pi}{2}$ $n \in I$	$\cot^{-1}x$	$\frac{-1}{1+x^2}$; $x \in R$

RULES FOR DIFFERENTIATION

$$\frac{d}{dx}(K(f(x)) = K\cdot\frac{d}{dx}(f(x)), \text{ where K is constant}$$

$$\frac{d}{dx}\{f(x) \pm g(x)\} = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$$

Product Rule: $\frac{d}{dx}\{f(x).g(x)\} = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))$

Quotient Rule: $\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$

Chain Rule: If y is a function of u, u is a function of v and v a function of x, then

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dv} \times \frac{dv}{dx}$$

Parametric differentiation: If x = P(t) , y = Q(t), where 't' is parameter then,

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{d}{dt}(Q(t))}{\frac{d}{dt}(P(t))} = \frac{Q'(t)}{P'(t)}$$

Differentiation of one function w.r.t. other function

$$\frac{d(f(x))}{d(g(x))} = \frac{\frac{d}{dx}(f(x))}{\frac{d}{dx}(g(x))} = \frac{f'(x)}{g'(x)}$$

Logarithmic differentiation: It is applicable in following cases:

→ $y = \underbrace{f_1.f_2.f_3.....f_n}_{}$ (All are functions of 'x') (product, divide or power form)

→ $y = (f(x))^{g(x)}$

* Take log on both sides and then differentiate.