Past Year JEE Questions

Questions

Quetion: 01

If $f(x) = \begin{cases} \frac{1}{|x|} & ; \ |x| \geq 1 \\ ax^2 + b & ; \ |x| < 1 \end{cases}$ is differentiable at every point of the domain, then the values of a

and b are respectively :

A. $\frac{1}{2}, \frac{1}{2}$

B. $\frac{1}{2}, -\frac{3}{2}$

C. $\frac{5}{2}, -\frac{3}{2}$

D. $-\frac{1}{2}, \frac{3}{2}$

Solutions

Solution: 01

Explanation

$f(x) = \begin{cases} \frac{1}{|x|}, & |x| \geq 1 \\ ax^2 + b, & |x| < 1 \end{cases}$

$= \begin{cases} -\frac{1}{x}; & x \leq -1 \\ ax^2 + b; & -1 < x < 1 \\ \frac{1}{x}; & x \geq 1 \end{cases}$

As f(x) is differentiable so it is also continuous,

at x = 1,

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^+} f(x)$

$\Rightarrow a + b = \frac{1}{1}$

$\Rightarrow a + b = 1$ ...... (1)

As f(x) is differentiable, so at x = 1

L.H.D. = R.H.D.

$\Rightarrow 2ax = -\frac{1}{x^2}$

$\Rightarrow 2a = -1$

$\Rightarrow a = -\frac{1}{2}$

From (1), $b = \frac{3}{2}$