# Differentiability - Class XII

## Past Year JEE Questions

## Questions

### Quetion: 01

The function

$f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1}x, & |x| \le 1 \\ \frac{1}{2}(|x| - 1), & |x| > 1 \end{cases}$ is :

A. continuous on R–{–1} and differentiable on R–{–1, 1}
B. both continuous and differentiable on R–{1}
C. both continuous and differentiable on R–{–1}
D. continuous on R–{1} and differentiable on R–{–1, 1}

## Solutions

### Solution: 01

#### Explanation

$$f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1}x, & x \in [-1, 1] \\ \frac{1}{2}(x - 1), & x > 1 \\ \frac{1}{2}(-x - 1), & x < -1 \end{cases}$$

At x = 1

L.H.L = $\lim_{x\to 1^-}\left(\frac{\pi}{4} + \tan^{-1}x\right) = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

f(1) = $\frac{\pi}{4} + \tan^{-1}x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

R.H.L = $\lim_{x\to 1^+}\left(\frac{1}{2}(x - 1)\right)$ = 0

As L.H.L $\neq$ R.H.L so function is discontinuous $\Rightarrow$ non differentiable.

At x = -1

L.H.L = $\lim_{x\to -1^-}\left(\frac{1}{2}(-x - 1)\right) = \frac{1}{2}(-(-1) - 1)$ = 0

f(-1) = $\frac{\pi}{4} + \tan^{-1}(-1) = \frac{\pi}{4} - \frac{\pi}{4}$ = 0

R.H.L = $\lim_{x\to -1^+}\left(\frac{\pi}{4} + \tan^{-1}x\right)$

= $\frac{\pi}{4} + \tan^{-1}(-1) = \frac{\pi}{4} - \frac{\pi}{4}$ = 0

As L.H.L = f(-1) = R.H.L so function is continuous.

$$f'(x) = \begin{cases} \frac{1}{1+x^2}, & x \in [-1, 1] \\ \frac{1}{2}, & x > 1 \\ -\frac{1}{2}, & x < -1 \end{cases}$$

For differentiability at x = –1

L.H.D = $-\frac{1}{2}$

R.H.D. = $\frac{1}{2}$

So, non differentiable at x = –1