

Exemplar Problems

Derivatives

29.

$$\sin \sqrt{x} + \cos^2 \sqrt{x}$$

Solution:

Let $y = \sin \sqrt{x} + \cos^2 \sqrt{x}$

Differentiating both sides w.r.t. x

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}(\sin \sqrt{x}) + \frac{d}{dx}(\cos^2 \sqrt{x}) \\ &= \cos \sqrt{x} \cdot \frac{d}{dx}(\sqrt{x}) + 2 \cos \sqrt{x} \cdot \frac{d}{dx}(\cos \sqrt{x}) \\ &= \cos \sqrt{x} \cdot \frac{1}{2\sqrt{x}} + 2 \cos \sqrt{x} (-\sin \sqrt{x}) \cdot \frac{d}{dx} \sqrt{x} \\ &= \frac{1}{2\sqrt{x}} \cdot \cos \sqrt{x} - 2 \cos \sqrt{x} \cdot \sin \sqrt{x} \cdot \frac{1}{2\sqrt{x}} \\ &= \frac{\cos \sqrt{x}}{2\sqrt{x}} - \frac{\sin 2\sqrt{x}}{2\sqrt{x}} \end{aligned}$$

Thus, $\frac{dy}{dx} = \frac{\cos \sqrt{x}}{2\sqrt{x}} - \frac{\sin 2\sqrt{x}}{2\sqrt{x}}$.