

Exemplar Problems

Derivatives

20.

$$f(x) = \begin{cases} x[x], & \text{if } 0 \leq x < 2 \\ (x-1)x, & \text{if } 2 \leq x < 3 \end{cases} \quad \text{at } x = 2.$$

Solution:

We know that, a function f is differentiable at a point 'a' in its domain if

$$Lf'(c) = Rf'(c)$$

$$\text{where } Lf'(c) = \lim_{h \rightarrow 0} \frac{f(a-h) - f(a)}{-h} \quad \text{and}$$

$$Rf'(c) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$\text{Here, at } x = 2 \quad f(x) = \begin{cases} x[x], & \text{if } 0 \leq x < 2 \\ (x-1)x, & \text{if } 2 \leq x < 2 \end{cases} \quad \text{at } x = 2.$$

$$\begin{aligned} Lf'(c) &= \lim_{h \rightarrow 0} \frac{f(2-h) - f(2)}{-h} = \lim_{h \rightarrow 0} \frac{(2-h)[2-h] - (2-1)2}{-h} \\ &= \lim_{h \rightarrow 0} \frac{(2-h) \cdot 1 - 2}{-h} \quad [\because [2-h] = 1] \end{aligned}$$

$$= \lim_{h \rightarrow 0} \frac{2-h-2}{-h} = 1$$

$$\begin{aligned} Rf'(c) &= \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(2+h-1)(2+h) - (2-1) \cdot 2}{h} \\ &= \lim_{h \rightarrow 0} \frac{(1+h)(2+h) - 2}{h} = \lim_{h \rightarrow 0} \frac{2+h+2h+h^2-2}{h} \\ &= \lim_{h \rightarrow 0} \frac{3h+h^2}{h} = \lim_{h \rightarrow 0} \frac{h(3+h)}{h} = 3 \end{aligned}$$

$$Lf'(2) \neq Rf'(2)$$

Therefore, $f(x)$ is not differentiable at $x = 2$.