

Exemplar Problems

Derivatives

1. Examine the continuity of the function $f(x) = x^3 + 2x^2 - 1$ at $x = 1$

Solution:

We know that, $y = f(x)$ will be continuous at $x = a$ if,

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$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a^+} f(x)$$

Given: $f(x) = x^3 + 2x^2 - 1$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} (1+h)^3 + 2(1+h)^2 - 1 = 1 + 2 - 1 = 2$$

$$\begin{aligned} \lim_{x \rightarrow 1} f(x) &= (1)^3 + 2(1)^2 - 1 \\ &= 1 + 2 - 1 = 2 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 1^+} f(x) &= \lim_{h \rightarrow 0} (1+h)^3 + 2(1+h)^2 - 1 \\ &= 1 + 2 - 1 = 2 \end{aligned}$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1^+} f(x) = 2.$$

Hence, $f(x)$ is continuous at $x = 1$.

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