Class Notes
Linear Programming

What is Linear Programming?

Linear programming (LP) or **Linear Optimisation** may be defined as the problem of maximizing or minimizing a linear function that is subjected to linear constraints. The constraints may be equalities or inequalities. The optimisation problems involve the calculation of profit and loss. Linear programming problems are an important class of optimisation problems, that helps to find the feasible region and optimise the solution in order to have the highest or lowest value of the function.

In other words, linear programming is considered as an optimization method to maximize or minimize the objective function of the given mathematical model with the set of some requirements which are represented in the linear relationship. The main aim of the linear programming problem is to find the optimal solution.

Linear programming is the method of considering different inequalities relevant to a situation and calculating the best value that is required to be obtained in those conditions. Some of the assumptions taken while working with linear programming are:

- The number of constraints should be expressed in the quantitative terms
- The relationship between the constraints and the objective function should be linear
- The linear function (i.e., objective function) is to be optimised

Components of Linear Programming

The basic components of the LP are as follows:

- Decision Variables
- Constraints
- Data
- Objective Functions

Characteristics of Linear Programming

The following are the five characteristics of the linear programming problem:

Constraints – The limitations should be expressed in the mathematical form, regarding the resource.

Objective Function – In a problem, the objective function should be specified in a quantitative way.

Linearity – The relationship between two or more variables in the function must be linear. It means that the degree of the variable is one.

Finiteness – There should be finite and infinite input and output numbers. In case, if the function has infinite factors, the optimal solution is not feasible.

Non-negativity – The variable value should be positive or zero. It should not be a negative value.

Decision Variables – The decision variable will decide the output. It gives the ultimate solution of the problem. For any problem, the first step is to identify the decision variables.

Linear Programming Concepts

In this lecture, linear programming concepts covered are:

- Introduction
- Linear programming problems and mathematical formulation
- A mathematical formulation of the problems
- Graphical method of solving linear programming problems
- Different types of linear programming problems

Different Type of Linear Programming Problems

The various types of problem in linear programming problem included in class 12 concepts. They are:

(i) Manufacturing problem- Here we maximize the profit with the help of minimum utilization of the resource.

(ii) Diet Problem- We determine the number of different nutrients in a diet to minimize the cost of manufacturing.

(iii) Transportation problem- Here we determine the schedule to find the cheapest way of transporting a product at minimum time.

Terms of Linear Programming Problem

There exist specific terms (or terminology) while constructing and solving linear programming problems. Let us define some of the important terms which we shall be using here.

Objective function: A linear function of the form $Z = ax + by$, where a and b are constant, which has to be minimized or maximized is called a linear objective function.
Consider an example, $Z = 175x + 150y$.
This is a linear objective function. The variables x and y are called decision variables.

Constraints: The linear inequalities or equations or restrictions on the variables of LPP (linear programming problem) are called constraints. The conditions $x \geq 0$, $y \geq 0$ are called non-negative restrictions.
For example, $5x + y \leq 100$; $x + y \leq 60$ are constraints.

Optimization problem: A problem which seeks to maximize or minimize a linear function (say of two variables x and y) subject to certain constraints as determined by a set of linear inequalities is called an optimization problem. Linear programming problems are special types of optimization problems.

Feasible region: The common region determined by all the given constraints including non-negative constraints ($x \geq 0$, $y \geq 0$) of a linear programming problem is called the feasible region (or solution region) for the problem. The region other than feasible is called an infeasible region.

Feasible solutions: These are the points within and on the boundary of the feasible region represent feasible solutions of the constraints. Any point outside the feasible region is called an infeasible solution.

Optimal (or feasible) solution: Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution.

Note:

- A corner point of a feasible region is the point of intersection of two boundary lines, which form the region.
- A feasible region of a given system of linear inequalities is said to be bounded if it can be enclosed within a circle. Otherwise, it is unbounded. Unbounded means that the feasible region may extend indefinitely in any direction.

Question: Solve the following linear programming problem graphically:

Minimize Z = 200 x + 500 y

subject to the constraints:

$x + 2y \geq 10$

$3x + 4y \leq 24$

$x \geq 0, y \geq 0$

Solution:

Given objective function is:

Minimize Z = 200 x + 500 y ....(i)

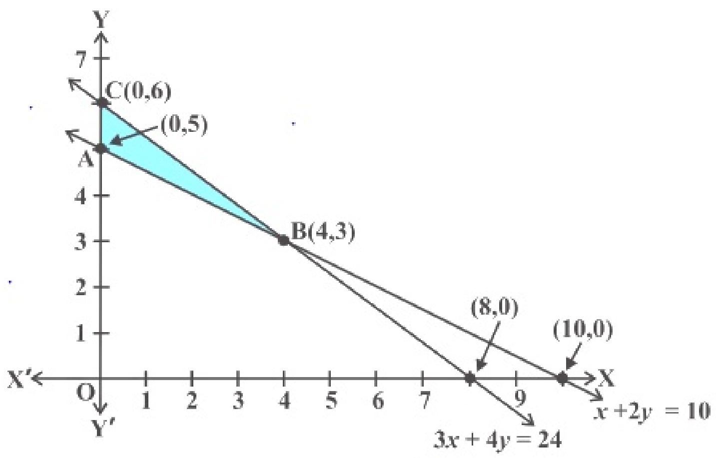
Constraints are:

$x + 2y \geq 10$ ....(ii)

$3x + 4y \leq 24$ ....(iii)

$x \geq 0, y \geq 0$ ....(iv)

The graph of these inequalities is:



The shaded region in the figure (above graph) is the feasible region ABC determined by the system of constraints (ii), (iii) and (iv), which is bounded.

The coordinates of corner points of this (feasible or shaded) region say A, B and C are (0, 5), (4, 3) and (0, 6) respectively.

Now, let us evaluate the value of Z = 200x + 500y at these points.

Corner point	Corresponding value of Z
(0, 5)	200 × 0 + 500 × 5 = 0 + 2500 = 2500
(4, 3)	200 × 4 + 500 × 3 = 800 + 1500 = 2300 (minimum)
(0, 6)	200 × 0 + 500 × 6 = 0 + 3000 = 3000

Hence, the minimum value of Z is 2300 at the point (4, 3).