

- Direction cosines of a line are the cosines of the angles made by the line with the positive directions of the coordinate axes.

- If l, m, n are the direction cosines of a line, then

$$l^2 + m^2 + n^2 = 1.$$

- Direction cosines of a line joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are

$$\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ}$$

Where, $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

- Direction ratios of a line are the numbers which are proportional to the direction cosines of a line.
- If l, m, n are the direction cosines and a, b, c are the direction ratios of a line

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}; m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

- Vector equation of a line that passes through the given point whose position vector is \vec{a} and parallel to a given vector \vec{b} is $\vec{r} = \vec{a} + \lambda\vec{b}$.

- Cartesian equation of a line that passes through two Points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

- Distance between parallel lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}$ is

$$\frac{|\vec{b} \times (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}|}$$

- Equation of a plane which is at a distance of d from the origin and the direction cosines of the normal to the plane as l, m, n is $lx + my + nz = d$.
- Equation of a plane that cuts the coordinates axes at $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$ is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

- Two planes $a_1 x + b_1 y + c_1 z + d_1 = 0$ and $a_2 x + b_2 y + c_2 z + d_2 = 0$ are coplanar if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

- The distance of a point whose position vector is \vec{a} from the plane $\vec{r} \cdot \hat{n} = d$ is $|d - \vec{a} \cdot \hat{n}|$.