Question 9: In R^3, consider the planes P_1: y = 0 and P_2: x+z = 1. Let P_3 be a plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point (0, 1, 0) from P_3 is 1 and the distance of a point (α, β, γ) from P_3 is 2, then which of the following relation is (are) true?

(a) $2\alpha + \beta + 2\gamma + 2 = 0$

(b) $2\alpha - \beta + 2\gamma + 4 = 0$

(c) $2\alpha + \beta - 2\gamma - 10 = 0$

(d) $2\alpha - \beta + 2\gamma - 8 = 0$

Solution:

Let P_3 be $P_2 + \lambda P_1 = 0$

=> $(x+z-1) + \lambda y = 0$

=> $x+\lambda y+z-1 = 0$

Distance of the point (0, 1, 0) from P_3 is $|(\lambda-1)/\sqrt{(2+\lambda^2)}| = 1$

=> $(\lambda-1)^2 = (2+\lambda^2)$

=> $-2\lambda + 1 = 2$

=> $-2\lambda = 1$

=> $\lambda = -1/2$

Distance of point (α, β, γ) from P_3: $|(\alpha+\lambda\beta + \gamma-1) /\sqrt{(2+\lambda^2)}| = 2$

$|(\alpha-(\beta/2) + \gamma-1) /(3/2)| = \pm 2$

$\alpha- (\beta/2) + \gamma-1 = \pm 3$

=> $2\alpha- \beta + 2\gamma-2 = \pm 6$

=> $2\alpha- \beta + 2\gamma-8 = 0$ or $2\alpha- \beta + 2\gamma + 4 = 0$

Hence option b and d are the answers.