Let P be a plane passing through the points (1, 0, 1), (1, −2, 1) and (0, 1, −2). Let a vector $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ be such that $\vec{a}$ is parallel to the plane P, perpendicular to $(\hat{i} + 2\hat{j} + 3\hat{k})$ and $\vec{a}.\ (\hat{i} + \hat{j} + 2\hat{k}) = 2$, then $(\alpha - \beta + \gamma)^2$ equals _____.

Answer

Correct Answer is **81**

Explanation

Equation of plane :

$$\begin{vmatrix} x-1 & y-0 & z-1 \\ 1-1 & 2 & 1-1 \\ 1-0 & 0-1 & 1+2 \end{vmatrix} = 0$$

$\Rightarrow 3x - z - 2 = 0$

$\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ || to 3x − z − 2 = 0

$\Rightarrow 3\alpha - 8 = 0$ ..... (1)

$\vec{a} \perp \hat{i} + \hat{j} + 3\hat{k}$

$\Rightarrow \alpha + 2\beta + 38 = 0$ ..... (2)

$\overrightarrow{a}.(\hat{i}+\hat{j}+2\widehat{k})=0$

$\Rightarrow \alpha + \beta + 28 = 2$ ..... (3_)

On solving 1, 2 & 3

$\alpha = 1, \beta = -5, 8 = 3$

So, $(\alpha - \beta + 8) = 81$