

Question : If the plane $2ax - 3ay + 4az + 6 = 0$ passes through the midpoint of the line joining the centres of the spheres $x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$, then a equals _____.

Solution:

$$S1 = x^2 + y^2 + z^2 + 6x - 8y - 2z = 13, C1 = (-3, 4, 1)$$

$$S2 = x^2 + y^2 + z^2 - 10x + 4y - 2z = 8, C2 = (5, -2, 1)$$

So, mid point of $C1 C2$ (say P) = $P($

$$\frac{5-3}{2}, \frac{4-2}{2}, \frac{1+1}{2})$$

$$= P(1, 1, 1)$$

Now the plane $2ax - 3ay + 4az + 6 = 0$ passes through the point P ,

$$\text{So, } 2a(1) - 3a(1) + 4a(1) + 6 = 0 = 2a - 3a + 4a + 6 = 0$$

$$3a + 6 = 0$$

$$3a = -6$$

$$\Rightarrow a = -2$$