

6. Prove that the lines  $x = py + q$ ,  $z = ry + s$  and  $x = p'y + q'$ ,  $z = r'y + s'$  are perpendicular if  $pp' + rr' + 1 = 0$ .

Sol. We have line  $x = py + q$ ,  $z = ry + s$

$$\Rightarrow y = \frac{x-q}{p} \text{ and } y = \frac{z-s}{r}$$

$$\Rightarrow \frac{x-q}{p} = \frac{y}{1} = \frac{z-s}{r} \quad \text{(i)}$$

Similarly line  $x = p'y + q'$ ,  $z = r'y + s'$

$$\Rightarrow \frac{x-q'}{p'} = \frac{y}{1} = \frac{z-s'}{r'} \quad \text{(ii)}$$

Line (i) is parallel to the vector  $p\hat{i} + \hat{j} + r\hat{k}$ .

Line (ii) is parallel to the vector  $p'\hat{i} + \hat{j} + r'\hat{k}$ .

Lines are perpendicular,

$$\therefore (p\hat{i} + \hat{j} + r\hat{k}) \cdot (p'\hat{i} + \hat{j} + r'\hat{k})$$

$$\therefore pp' + 1 + rr' = 0.$$