# 1 Properties of Determinants

### Property 1

If a determinant has all the elements zero in any row (or column) then its values is zero.

$$\Delta = \begin{vmatrix} 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0$$

### Property 2

Determinant of a diagonal matrix is given by product of its diag entries.

$$\Delta = \begin{vmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33}$$

### Property 3

Determinant of a upper or lower triagonal matrix is given by product of its diag entries.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33}$$

### Property 4

The value of a determinant remains unaltered; if the rows and columns are interchanged. Basically transpose of a matrix has same determinant.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

### Property 5

If any two adjacent rows (or columns) of a determinant be interchanged, the value of determinant is changed in sign only.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\Delta' = \begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Then we have $\Delta = -\Delta'$.

### Property 6

If a determinant has any two rows (or columns) identical or proportional, then its values is zero.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0$$

## Property 7

If all the elements of any row (or column) be multiplied by the same number, then the determinant is multiplied by that number.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\Delta^{'} = \begin{vmatrix} Ka_{11} & Ka_{12} & Ka_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{23} & a_{33} \end{vmatrix}$$

Then we have $\Delta^{'} = K\Delta$.

## Property 8

If each element of any row (or column) can be expressed as a sum of two terms then the determinant can be expressed as the sum of two determinants, that means

$$\Delta = \begin{vmatrix} x + a_{11} & y + a_{12} & z + a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} x & y & z \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

## Property 9

The value of determinant is not altered by adding to the elements of any row (or column) a constant multiple of the corresponding elements of any other row (or column).
**Exa:**
$R_1 \rightarrow R_1 + mR_2$ (change R1 as sum of R1 and m (R2).
$R_3 \rightarrow R_3 + nR_2$ (change R3 as sum of R3 and n (R2).

$$\Delta = \begin{vmatrix} a_{11} + ma_{21} & a_{12} + ma_{22} & a_{13} + ma_{23} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\Delta^{'} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} + na_{21} & a_{32} + na_{22} & a_{33} + na_{23} \end{vmatrix}$$

Then this property says, $\Delta = \Delta^{'}$
**NOTE:** All these properties can be proved just by doing step by step algebra of determinant calculations. Take it as an exercise to prove these results for general form of 3 x 3 matrix.