

Concepts and Formulas

Properties of Determinants:

(i) The value of the determinant remains unchanged, if rows are changed into columns and columns are changed into rows e.g.,

$$|A'| = |A|$$

(ii) If $A = [a_{ij}]_{n \times n}$, $n > 1$ and B be the matrix obtained from A by interchanging two of its rows or columns, then

$$\det(B) = -\det(A)$$

(iii) If two rows (or columns) of a square matrix A are proportional, then $|A| = 0$.

(iv) $|B| = k|A|$, where B is the matrix obtained from A , by multiplying one row (or column) of A by k .

(v) $|kA| = kn|A|$, where A is a matrix of order $n \times n$.

(vi) If each element of a row (or column) of a determinant is the sum of two or more terms, then the determinant can be expressed as the sum of two or more determinants, e.g.,

$$\begin{vmatrix} a_1 + a_2 & b & c \\ p_1 + p_2 & q & r \\ u_1 + u_2 & v & \end{vmatrix} = \begin{vmatrix} a_1 & b & c \\ p_1 & q & r \\ u_1 & v & \end{vmatrix} + \begin{vmatrix} a_2 & b & c \\ p_2 & q & r \\ u_2 & v & \end{vmatrix}$$

(vii) If the same multiple of the elements of any row (or column) of a determinant are added to the corresponding elements of any other row (or column), then the value of the new determinant remains unchanged, e.g.,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} + ka_{31} & a_{12} + ka_{32} & a_{13} + ka_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

(viii) If each element of a row (or column) of a determinant is zero, then its value is zero.

(ix) If any two rows (columns) of a determinant are identical, then its value is zero.