Related Questions with Solutions

Questions

Quetion: 01

The value of the determinant

$$\begin{vmatrix} -\left(2^5+1\right)^2 & 2^{10}-1 & \frac{1}{2^5-1} \\ 2^{10}-1 & -\left(2^5-1\right)^2 & \frac{1}{2^5+1} \\ \frac{1}{2^5-1} & \frac{1}{2^5+1} & -\frac{1}{\left(2^{10}-1\right)^2} \end{vmatrix}$$

is -

A. 0

B. 1

C. 2

D. 4

Solutions

Solution: 01

Taking $2^5+1=a$ and $2^5-1=b$, then $2^{10}-1=\left(2^5+1\right)\left(2^5-1\right)=ab$, therefore the given determinant equals.

$$\Delta=\begin{vmatrix} -a^2 & ab & \frac{1}{b} \\ ab & -b^2 & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{a} & -\frac{1}{a^2b^2} \end{vmatrix}$$

Multiplying R_1 with b, R_2 with a and R_3 with $\mathrm{a}^2\ \mathrm{b}^2$

$$\Delta=\frac{1}{a^3b^3}\begin{vmatrix} -ba^2 & ab^2 & 1 \\ ba^2 & -ab^2 & 1 \\ ba^2 & ab^2 & -1 \end{vmatrix}$$

$\mathrm{R}_1 \rightarrow \mathrm{R}_1+\mathrm{R}_2$

$$=\frac{1}{a^3b^3}\begin{vmatrix} 0 & 0 & 2 \\ ba^2 & -ab^2 & 1 \\ ba^2 & ab^2 & -1 \end{vmatrix}$$

Expanding along R_1

$$=\frac{2}{a^3b^3}\cdot\begin{vmatrix} ba^2 & -ab^2 \\ ba^2 & ab^2 \end{vmatrix}$$

$$=\frac{2}{a^3b^3}\left(a^3b^3\right)\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}=4$$

Correct Options

Answer:01

Correct Options: D