

Determinants - Class XII

Past Year JEE Questions

Questions

Question: 01

A value of $\theta \in (0, \frac{\pi}{3})$, for which

$$\begin{vmatrix} 1 + \cos^2\theta & \sin^2\theta & 4 \cos 6\theta \\ \cos^2\theta & 1 + \sin^2\theta & 4 \cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1 + 4 \cos 6\theta \end{vmatrix} = 0, \text{ is :}$$

- A. $\frac{\pi}{18}$
- B. $\frac{\pi}{9}$
- C. $\frac{1}{24}\pi$
- D. $\frac{1}{36}\pi$

Solutions

Solution: 01

Explanation

$$\begin{vmatrix} 1 + \cos^2\theta & \sin^2\theta & 4 \cos 6\theta \\ \cos^2\theta & 1 + \sin^2\theta & 4 \cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1 + 4 \cos 6\theta \end{vmatrix} = 0$$

$$R_1 \rightarrow R_1 - R_2, R_2 \rightarrow R_2 - R_3$$

$$\Rightarrow \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos^2\theta & \sin^2\theta & 1 + 4 \cos 6\theta \end{vmatrix} = 0$$

$$C_2 \rightarrow C_2 + C_1$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ \cos^2\theta & 1 & 1 + 4 \cos 6\theta \end{vmatrix} = 0$$

$$\Rightarrow 1 + 4 \cos 6\theta + 1 = 0$$

$$\Rightarrow 2 \cos 6\theta = -1 \Rightarrow \cos 6\theta = -\frac{1}{2} = \cos \frac{2\pi}{3}$$

$$\Rightarrow 6\theta = 2n\pi \pm \frac{2\pi}{3}$$

$$\Rightarrow \theta = \frac{n\pi}{3} \pm \frac{\pi}{9}, n \in \mathbb{Z}$$

$$\Rightarrow \theta = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}$$