

Exemplar Problems

Determinants

13. If $\Delta = 0$, then find values of x .

$$\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix}$$

Solution:

Given,

$$\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$$

[Applying $R_1 \rightarrow R_1 + R_2 + R_3$], we have

$$\Rightarrow \begin{vmatrix} 12+x & 12+x & 12+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$$

Now,

[Taking $(12+x)$ common from R_1]

$$\Rightarrow (12+x) \begin{vmatrix} 1 & 1 & 1 \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$$

Next,

[Applying $C_1 \rightarrow C_1 - C_3$ and $C_2 \rightarrow C_2 - C_3$]

$$\Rightarrow (12+x) \begin{vmatrix} 0 & 0 & 1 \\ 0 & -2x & 4+x \\ 2x & 2x & 4-x \end{vmatrix} = 0$$

$$\Rightarrow (12+x)(0 - (-2x)(2x)) = 0$$

$$(12+x)(4x^2) = 0$$

Hence, $x = -12, 0$

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[Applying $R_1 \rightarrow R_1 + R_2 + R_3$], we have

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Now,

[Taking $(12+x)$ common from R_1]

$$\Rightarrow (12+x) \begin{vmatrix} 1 & 1 & 1 \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$$

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[Applying $C_1 \rightarrow C_1 - C_3$ and $C_2 \rightarrow C_2 - C_3$]

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