14. If $a_1, a_2, a_3, \ldots, a_r$ are in G.P., then prove that the determinant

$$\begin{vmatrix} a_{r+1} & a_{r+5} & a_{r+9} \\ a_{r+7} & a_{r+11} & a_{r+15} \\ a_{r+11} & a_{r+17} & a_{r+21} \end{vmatrix}$$

is independent of *r*.

Solution:

We know that,

$$a_{r+1} = AR^{(r+1)-1} = AR^r;$$

where a_r = rth term of G.P.,

A = First term of G.P.

and R = Common ratio of G.P.

Now, $$\begin{vmatrix} a_{r+1} & a_{r+5} & a_{r+9} \\ a_{r+7} & a_{r+11} & a_{r+15} \\ a_{r+11} & a_{r+17} & a_{r+21} \end{vmatrix} = \begin{vmatrix} AR^r & AR^{r+4} & AR^{r+8} \\ AR^{r+6} & AR^{r+10} & AR^{r+14} \\ AR^{r+10} & AR^{r+16} & AR^{r+20} \end{vmatrix}$$

[Taking AR^r, AR^{r+6} and AR^{r+10} common from R_1, R_2 and R_3, respectively]

$$= AR^r \cdot AR^{r+6} \cdot AR^{r+10} \begin{vmatrix} 1 & AR^4 & AR^8 \\ 1 & AR^4 & AR^8 \\ 1 & AR^6 & AR^{10} \end{vmatrix}$$

$= 0$ [As R_1 and R_2 are identical]

Hence, the determinant is independent of r.