12. Find the value of q satisfying

$$\begin{bmatrix} 1 & 1 & \sin 3\theta \\ -4 & 3 & \cos 2\theta \\ 7 & -7 & -2 \end{bmatrix} = 0 .$$

Solution:

Given,

$$\begin{vmatrix} 1 & 1 & \sin 3\theta \\ -4 & 3 & \cos 2\theta \\ 7 & -7 & -2 \end{vmatrix} = 0$$

On expanding along C_3, we have

$\sin 3\theta \times (28 - 21) - \cos 2\theta \times (-7 - 7) - 2(3 + 4) = 0$

$7 \sin 3\theta + 14 \cos 2\theta - 14 = 0$

$\sin 3\theta + 2 \cos 2\theta - 2 = 0$

$(3 \sin \theta - 4 \sin^3 \theta) + 2(1 - 2 \sin^2 \theta) - 2 = 0$

$4 \sin^3\theta - 4 \sin^2 \theta + 3 \sin \theta = 0$

$\sin \theta(4 \sin^2 \theta - 4 \sin \theta + 3) = 0$

$\sin \theta(4 \sin^2 \theta - 6 \sin \theta + 2 \sin \theta + 3) = 0$

$\sin \theta(2 \sin \theta + 1)(2 \sin \theta - 3) = 0$

$\sin \theta = 0$ or $\sin \theta = -1/2$ or $\sin \theta = 3/2$

$$\theta = n\pi \text{ or } \theta = m\pi + (-1)^n \left(-\frac{\pi}{6}\right); m, n \in Z$$

$\sin \theta = \dfrac{-3}{2}$ is not possible