Past Year JEE Questions

Questions

Quetion: 01

If $A = \begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}$ and $\det\left(A^2 - \frac{1}{2}I\right) = 0$, then a possible value of α is :

A. $\frac{\pi}{4}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{3}$

Solutions

Solution: 01

Explanation

$$A^2 = \begin{bmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{bmatrix} \begin{bmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{bmatrix} = \begin{bmatrix} \sin^2\alpha & 0 \\ 0 & \sin^2\alpha \end{bmatrix}$$

$$A^2 - \frac{1}{2}I = \begin{bmatrix} \sin^2\alpha & 0 \\ 0 & \sin^2\alpha \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \sin^2\alpha - \frac{1}{2} & 0 \\ 0 & \sin^2\alpha - \frac{1}{2} \end{bmatrix}$$

Given, $\left|A^2 - \frac{1}{2}I\right| = 0$

$$\Rightarrow \begin{vmatrix} \sin^2\alpha - \frac{1}{2} & 0 \\ 0 & \sin^2\alpha - \frac{1}{2} \end{vmatrix} = 0$$

$$\Rightarrow \left(\sin^2\alpha - \frac{1}{2}\right)^2 = 0$$

$$\Rightarrow \sin^2\alpha = \frac{1}{2} \Rightarrow \sin\alpha = \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$$

$$\therefore \alpha = \frac{\pi}{4}$$