

Exemplar Problems

Determinants

5.

$$\begin{vmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix}$$

Solution:

$$\begin{aligned}
 \text{Given, } & \begin{vmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix} \\
 &= \begin{vmatrix} 3x+4 & 3x+4 & 3x+4 \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix} \quad [\text{Applying } R_1 \rightarrow R_1 + R_2 + R_3] \\
 &= (3x+4) \begin{vmatrix} 1 & 1 & 1 \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix} \\
 &= (3x+4) \begin{vmatrix} 0 & 0 & 1 \\ -4 & 4 & x \\ 0 & -4 & x+4 \end{vmatrix} = 16(3x+4) \\
 &\quad [\text{Applying } C_1 \rightarrow C_1 - C_2, C_2 \rightarrow C_2 - C_3]
 \end{aligned}$$