

## Exemplar Problems

### Determinants

2.

$$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$$

Solution:

Given, 
$$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$$

[Applying  $C_1 \rightarrow C_1 + C_2 + C_3$ ]

$$\begin{aligned} &= \begin{vmatrix} a+x+y+z & y & z \\ a+x+y+z & a+y & z \\ a+x+y+z & y & a+z \end{vmatrix} \\ &= (a+x+y+z) \begin{vmatrix} 1 & y & z \\ 1 & a+y & z \\ 1 & y & a+z \end{vmatrix} \end{aligned}$$

[Applying  $R_2 \rightarrow R_2 - R_1$  and  $R_3 \rightarrow R_3 - R_1$ ]

$$\begin{aligned} &= (a+x+y+z) \begin{vmatrix} 1 & y & z \\ 0 & a & 0 \\ 0 & 0 & a \end{vmatrix} \\ &= (a+x+y+z) \begin{vmatrix} a & 0 \\ 0 & a \end{vmatrix} = a^2(a+z+x+y) \end{aligned}$$

Given, 
$$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$$

[Applying  $C_1 \rightarrow C_1 + C_2 + C_3$ ]

$$\begin{aligned} &= \begin{vmatrix} a+x+y+z & y & z \\ a+x+y+z & a+y & z \\ a+x+y+z & y & a+z \end{vmatrix} \\ &= (a+x+y+z) \begin{vmatrix} 1 & y & z \\ 1 & a+y & z \\ 1 & y & a+z \end{vmatrix} \end{aligned}$$

[Applying  $R_2 \rightarrow R_2 - R_1$  and  $R_3 \rightarrow R_3 - R_1$ ]

$$\begin{aligned} &= (a+x+y+z) \begin{vmatrix} 1 & y & z \\ 0 & a & 0 \\ 0 & 0 & a \end{vmatrix} \\ &= (a+x+y+z) \begin{vmatrix} a & 0 \\ 0 & a \end{vmatrix} = a^2(a+z+x+y) \end{aligned}$$