

## द्विपद प्रमेय

$$n! \text{ या } n! = 1 \times 2 \times 3 \times 4 \times \dots \times n$$

$$n! = n \times (n-1)! = n \times (n-1) \times (n-2)! \dots$$

$$\frac{n!}{n!} = \frac{n!}{n!} \quad 1 = 1$$

$$n=0 \Rightarrow \frac{0!}{0!} = \frac{1}{1} = 1 = \infty$$

$$n=-1 \Rightarrow \frac{(-1)!}{(-1)!} = \frac{(-1)!}{(-1)!} = \frac{\infty}{-1} = -\infty$$

$$\boxed{\begin{aligned} nC_r &= nC_{n-r} \\ nC_r + nC_{r-1} &= {}^{n+1}C_r \end{aligned}}$$

$$\begin{aligned} nC_r &= \binom{n}{r} = C(n, r) \\ n &\geq 0, 0 \leq r \leq n \end{aligned}$$

$${}^nC_0 = 1 \quad {}^nC_n = \frac{n!}{n! \cdot 0!} = 1$$

$${}^nC_n = \frac{n!}{n! \cdot 0!} = \frac{n!}{n! \cdot 1} = 1$$

$${}^nC_1 = \frac{n!}{1! \cdot (n-1)!} = n$$

$$nPr = nCr \times r!$$

प्र

① जब  $n$  धनात्मक पूर्णांक है तब द्विपद प्रमेय :-

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \dots + {}^nC_{n-2} x^2 y^{n-2} + {}^nC_{n-1} x^1 y^{n-1} + {}^nC_n x^0 y^n$$

\*  $x$  व  $y$  की घातों का योग सदैव  $n$  रहता है

\* पदों की संख्या =  $n+1$

\*  ${}^nC_0, {}^nC_1, {}^nC_2, {}^nC_3, \dots, {}^nC_{n-2}, {}^nC_{n-1}, {}^nC_n$  = द्विपद गुणांक

\* प्रसार में दोनों तरफ से समान दूरी पर स्थित द्विपद गुणांकों के मान समान होते हैं



पारस्परिक विभुज

1	1	1	1	1	1
1	3	2	3	1	1
1	6	6	4	1	1
1	10	10	5	1	1
1	15	20	15	6	1
1	21	35	35	21	7
1	28	56	70	56	28
1	36	90	126	126	84
1	45	135	210	252	168
1	55	198	330	462	330
1	66	286	504	858	660
1	78	408	756	1485	1365
1	91	578	1162	2520	2520
1	105	816	1771	4284	4620
1	120	1140	2700	7560	9000
1	136	1570	4074	13122	16665
1	153	2142	6188	23094	31500
1	171	2916	9450	40110	59535
1	190	3960	14400	70560	110780
1	210	5382	21840	128700	210210
1	231	7308	33642	230940	424284
1	253	9840	51480	422400	859800
1	276	13200	77142	807300	1714635
1	300	17580	113400	1548840	3527160
1	325	24080	170100	3152840	7262655
1	351	32820	255240	6052200	15189585
1	378	45000	380100	11765100	31528400
1	406	61080	558000	22539000	63551400
1	435	82620	826200	43545600	132365100
1	465	111600	1237500	84452400	279415440
1	496	150000	1848000	164581200	591839280
1	528	200880	2757600	324624000	1270819200
1	561	268800	4101600	630516000	2721705600
1	595	360000	6048000	1246416000	5812320000
1	630	480000	8964000	2501400000	12621600000
1	666	636000	13320000	5052000000	27648000000
1	703	846000	19716000	10560000000	60480000000
1	741	1137600	29016000	22464000000	137280000000
1	780	1540800	42840000	48024000000	311040000000
1	820	2097600	64800000	102960000000	699600000000
1	861	2868000	97680000	214800000000	1548000000000
1	903	3936000	146400000	460800000000	3456000000000
1	946	5352000	218400000	992400000000	7728000000000
1	990	7296000	329400000	2124000000000	17016000000000
1	1035	10080000	500400000	4572000000000	37200000000000
1	1081	13920000	746800000	10008000000000	82080000000000
1	1128	19200000	1116000000	21840000000000	181440000000000
1	1176	26400000	1680000000	47520000000000	403200000000000
1	1225	36000000	2520000000	104400000000000	891000000000000
1	1275	48840000	3720000000	229200000000000	1980000000000000
1	1326	66600000	5496000000	506400000000000	4464000000000000
1	1378	91200000	8204000000	1128000000000000	10080000000000000
1	1431	124800000	12120000000	2520000000000000	23040000000000000
1	1485	171600000	17880000000	5544000000000000	54720000000000000
1	1540	235200000	26760000000	12480000000000000	127680000000000000
1	1596	321600000	39720000000	28080000000000000	302400000000000000
1	1653	438000000	58920000000	63360000000000000	705600000000000000
1	1711	594000000	86640000000	144000000000000000	1612800000000000000
1	1770	806400000	127200000000	324000000000000000	3720000000000000000
1	1830	1104000000	186000000000	720000000000000000	16128000000000000000
1	1891	1512000000	272400000000	1608000000000000000	36288000000000000000
1	1953	2064000000	399600000000	3600000000000000000	81216000000000000000
1	2016	2816000000	589200000000	8064000000000000000	181440000000000000000
1	2080	3840000000	866400000000	18000000000000000000	403200000000000000000
1	2145	5208000000	1272000000000	40320000000000000000	907200000000000000000
1	2211	7008000000	1860000000000	90720000000000000000	2030400000000000000000
1	2278	9408000000	2724000000000	203040000000000000000	4572000000000000000000
1	2346	12672000000	3996000000000	457200000000000000000	10224000000000000000000
1	2415	17280000000	5892000000000	1022400000000000000000	23040000000000000000000
1	2485	23616000000	8664000000000	2304000000000000000000	51840000000000000000000
1	2556	32280000000	12720000000000	5184000000000000000000	117120000000000000000000
1	2628	44064000000	18600000000000	11712000000000000000000	264000000000000000000000
1	2701	59840000000	27240000000000	26400000000000000000000	594000000000000000000000
1	2775	81600000000	39960000000000	59400000000000000000000	1336800000000000000000000
1	2850	111600000000	58920000000000	133680000000000000000000	3024000000000000000000000
1	2926	153600000000	86640000000000	302400000000000000000000	6840000000000000000000000
1	3003	210000000000	127200000000000	684000000000000000000000	15480000000000000000000000
1	3081	285600000000	186000000000000	1548000000000000000000000	35280000000000000000000000
1	3160	393600000000	272400000000000	3528000000000000000000000	79200000000000000000000000
1	3240	537600000000	399600000000000	7920000000000000000000000	178560000000000000000000000
1	3321	734400000000	589200000000000	17856000000000000000000000	403200000000000000000000000
1	3403	1008000000000	866400000000000	40320000000000000000000000	912000000000000000000000000
1	3486	1387200000000	1272000000000000	91200000000000000000000000	2073600000000000000000000000
1	3570	1920000000000	1860000000000000	207360000000000000000000000	4672000000000000000000000000
1	3655	2664000000000	2724000000000000	467200000000000000000000000	10512000000000000000000000000
1	3741	3696000000000	3996000000000000	1051200000000000000000000000	23808000000000000000000000000
1	3828	5088000000000	5892000000000000	2380800000000000000000000000	53760000000000000000000000000
1	3916	7008000000000	8664000000000000	5376000000000000000000000000	121440000000000000000000000000
1	4005	9648000000000	12720000000000000	12144000000000000000000000000	277440000000000000000000000000
1	4095	13296000000000	18600000000000000	27744000000000000000000000000	621600000000000000000000000000
1	4186	18336000000000	27240000000000000	62160000000000000000000000000	1411200000000000000000000000000
1	4278	25440000000000	39960000000000000	141120000000000000000000000000	3192000000000000000000000000000
1	4371	35360000000000	58920000000000000	319200000000000000000000000000	7224000000000000000000000000000
1	4465	48960000000000	86640000000000000	722400000000000000000000000000	16320000000000000000000000000000
1	4560	67200000000000	127200000000000000	1632000000000000000000000000000	36960000000000000000000000000000
1	4656	91200000000000	186000000000000000	3696000000000000000000000000000	83520000000000000000000000000000
1	4753	123600000000000	272400000000000000	8352000000000000000000000000000	189600000000000000000000000000000
1	4851	168000000000000	399600000000000000	18960000000000000000000000000000	428400000000000000000000000000000
1	4950	228000000000000	589200000000000000	42840000000000000000000000000000	974400000000000000000000000000000
1	5050	309600000000000	866400000000000000	97440000000000000000000000000000	2217600000000000000000000000000000
1	5151	419200000000000	1272000000000000000	22176000000000000000000000000000	5040000000000000000000000000000000
1	5253	576000000000000	1860000000000000000	50400000000000000000000000000000	1142400000000000000000000000000000
1	5356	792000000000000	2724000000000000000	11424000000000000000000000000000	2592000000000000000000000000000000
1	5460	1080000000000000	3996000000000000000	25920000000000000000000000000000	5892000000000000000000000000000000
1	5565	1464000000000000	5892000000000000000	58920000000000000000000000000000	13368000000000000000000000000000000
1	5671	1984000000000000	8664000000000000000	13368000000000000000000000000000	30240000000000000000000000000000000
1	5778	2688000000000000	12720000000000000000	30240000000000000000000000000000	68400000000000000000000000000000000
1	5886	3648000000000000	18600000000000000000	68400000000000000000000000000000	154800000000000000000000000000000000
1	5995	4944000000000000	27240000000000000000	15480000000000000000000000000000	352800000000000000000000000000000000
1	6105	6672000000000000	39960000000000000000	35280000000000000000000000000000	792000000000000000000000000000000000
1	6216	9072000000000000	58920000000000000000	79200000000000000000000000000000	178560000000000000000000000000000000
1	6328	12480000000000000	86640000000000000000	17856000000000000000000000000000	403200000000000000000000000000000000
1	6441	17280000000000000	127200000000000000000	40320000000000000000000000000000	912000000000000000000000000000000000
1	6555	23160000000000000	186000000000000000000	91200000000000000000000000000000	207360000000000000000000000000000000
1	6670	31040000000000000	272400000000000000000	20736000000000000000000000000000	467200000000000000000000000000000000
1	6786	41760000000000000	399600000000000000000	46720000000000000000000000000000	1051200000000000000000000000000000000
1	6903	56160000000000000	589200000000000000000	10512000000000000000000000000000</	



Ans 3  $(2 + \frac{x}{3})^n$   $x^7$  का गुणांक =  $x^8$  का गुणांक

$$T_{r+1} = T_{7+1} = {}^nC_7 (2)^{n-8} \left(\frac{x}{3}\right)^8$$

$${}^nC_7 2 \left(\frac{x}{3}\right)^7 = {}^nC_8 \left(\frac{x}{3}\right)^8$$

$$\frac{{}^nC_7}{{}^nC_8} = \frac{x}{3 \times 2} = \frac{x}{6}$$

$$\frac{7! (n-7)!}{6! (n-8)!} = \frac{x}{6} \quad 6 {}^nC_7 = {}^nC_8$$

$$7n - 49 = \frac{1}{6} \Rightarrow 7n = \frac{1}{6} + 49$$

$$7n = \frac{1+294}{6} = \frac{295}{6} = n$$

$$6 {}^nC_7 = {}^nC_8$$

$$6 \times \frac{n!}{7! (n-7)!} = \frac{n!}{8! (n-8)!}$$

$$n-7 = 48 \quad n = 55$$

Ans  $(x^2 - 3\sqrt{3})^{10}$

$$(x^2 - 3\sqrt{3})^n = T_{r+1} = (-1)^r {}^nC_r x^{n-r} (3\sqrt{3})^r$$

$$(-1)^r {}^nC_r (x^2)^{10-r} \left(\frac{3\sqrt{3}}{x^3}\right)^r$$

$$(-1)^r {}^nC_r x^{20-2r} \frac{(3\sqrt{3})^r}{x^{3r}}$$

$$(-1)^r {}^nC_r x^{20-5r} (3\sqrt{3})^r$$

$$20 - 5r = 0$$

$$r = 4 \quad \boxed{r = 4} \quad {}^{10}C_4 (3\sqrt{3})^4 \text{ Ans}$$

Q  $\log_a K = K$

$$Q \quad \left[ \sqrt[2]{\log_2 \sqrt{3^{x-1} + 1}} + \frac{1}{\sqrt[5]{\log_2 (3^{x-1} + 1)}} \right] \text{ के प्रसार में } x=7$$

$$\text{Ans} \quad \left[ (9^{x-1} + 1)^{1/2} + \frac{1}{(3^{x-1} + 1)^{1/5}} \right]^7$$

$$T_{r+1} = {}^7C_r \left\{ (9^{x-1} + 1)^{1/2} + (3^{x-1} + 1)^{-1/5} \right\}^7 \times \left\{ (9^{x-1} + 1)^{-1/2} \right\}^r$$

$$T_2 = {}^7C_1 \times 2 \times \frac{1}{3^{x-1} + 1} = 84$$

$$\frac{3^{x-1} + 1}{3^{x-1} + 1} = 84$$

$$x^2 + 7 = 4x + 4$$

$$x^2 - 4x + 3 = 0 \Rightarrow x^2 - 3x - x + 3 = 0$$

$$\frac{3^{x-1}}{3^{x-1}} = 1$$

$$x-1 = 1 \quad x = 2$$

Ans  $x = (2, 1)$  Ans

Q.  $(5^{1/2} + 7^{1/8})^{1024}$  के प्रसार में शून्यक पदों की संख्या ज्ञात कीजिए

$$T_{r+1} = {}^{1024}C_r \left(5^{1/2}\right)^{1024-r} \times \left(7^{1/8}\right)^r$$

$$r = 0, 1, 16, \dots, 1024 \quad \text{Ans}$$

$$T_n = {}^{1024}C_n = 0 + (n-1)d = 1024$$

$$n-1 = \frac{1024}{8} = 128$$

$$n = 129$$

अतः शून्यक पदों की संख्या = 129  
शून्यक पदों की संख्या =  $1025 - 129 = 896$



$(x+a)^n$  के प्रसार में अन्त में  $r$  वा पद ज्ञात करना

$$T_{r+1} = T_{n-r+2} (B)$$

Ex)  $(x+a)^n$  के प्रसार में अन्त से  $r$  वा पद वही है जो  $(a+x)^n$  के प्रसार में शुरू से  $r$  वा पद है।

$(x+a)^n$  के प्रसार में मध्य पद ज्ञात करना -

(i)  $n$  सम ही तो कुल पद  $(n+1)$  ही तो जो कि विषम है तब मध्य पद

$$= \left(\frac{n+2}{2}\right) \text{वां पद} = \left(\frac{n+1}{2}\right) \text{वां पद}$$

(ii)  $n$  विषम ही तो

$n$  विषम ही तब कुल पद  $(n+1)$  ही तो जो कि सम है तब मध्य पद

$$= \frac{n+1}{2} \text{वां पद व } \left(\frac{n+3}{2}\right) \text{वां पद}$$

तब इन दोनों पदों के द्विपद गुणांक समान ही तो।

\*  $(x_1+x_2+x_3+\dots+x_k)^n$  के प्रसार में कुल पदों की संख्या

$$= \binom{n+k-1}{k-1} \text{ ही तो जबकि } x_1, x_2, x_3, \dots, x_k \text{ अलग-अलग चर हैं}$$

Examp  $(x+y+z)^4 = \dots$   ${}^6C_2 = 15$  Ans

\*  $(x+a)^n + (x-a)^n$  के प्रसार में कुल पद:-

$$= \left(\frac{n}{2} + 1\right) \text{ पद}$$

$n$  विषम है तब  $= \frac{n+1}{2}$  पद

\*  $(x+a)^n - (x-a)^n$  के प्रसार में कुल पद

$$n \text{ सम है तब } = \frac{n}{2} \text{ पद}$$

$n$  विषम है तब  $= \frac{n+1}{2}$  पद

Examp  $(x+a)^{100} + (x-a)^{100}$  के प्रसार में पदों की संख्या

$$\text{Ans } \frac{100}{2} + 1 = 51 \text{ Ans}$$

\*  $(x-1)(x-2)(x-3)\dots(x-n)$  के प्रसार में  $x^{n-1}$  का

$$\text{गुणांक } = -\frac{n(n+1)}{2}$$

के प्रसार में  $x^{n-1}$  का

$$\text{गुणांक } = \frac{n(n+1)}{2}$$

\* किसी बहुपद में  $x$  का शून्यिक स्थान पर  $1$  रखने से जो मान प्राप्त होता है वह मान उस बहुपद के प्रसार में समस्त गुणांकों

पदों के गुणांकों का योग होता है

किसी द्विपद के प्रसार में समस्त द्विपद गुणांकों का योग  $2^n$  होता है।

\*  $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n$

$$\text{Put } x=1$$

$$2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n$$

$$\text{Ex } {}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \dots + {}^{20}C_{20} = 2^{20} \quad \text{--- (1)}$$

$$\text{Ex } {}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \dots + {}^{20}C_{10} + {}^{20}C_{11} + \dots + {}^{20}C_{20} = 2^{20} \quad \text{--- (2)}$$

$$\text{Subtract (1) from (2): } {}^{20}C_1 + {}^{20}C_3 + \dots + {}^{20}C_{19} = 2^{20} - 2^{20} = 0$$

$$\text{Ex } {}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \dots + {}^{20}C_{10} = 2^{20} - 2^{20} = 0$$

Put  $x = 1$

$$2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n \quad \text{--- (1)}$$

Put  $x = -1$

$$0 = {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \dots + (-1)^n {}^nC_n \quad \text{--- (2)}$$

Adding (1) + (2)

$$2^{n-1} = {}^nC_0 + {}^nC_2 + {}^nC_4 + \dots$$

Subtracting (1) - (2)

$$2^{n-1} = {}^nC_1 + {}^nC_3 + {}^nC_5 + \dots$$

$$\times (1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n \quad \text{--- (3)}$$

$$\times (x+1)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} + \dots + {}^nC_{n-1} x + {}^nC_n$$

$$\times (1+x)^n = ({}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n) ({}^nC_0 x^n + {}^nC_1 x^{n-1} + \dots + {}^nC_n)$$

Equating coefficients of  $x^{n-1}$ ,  $x^{n-2}$ , etc. equating coefficients

$$2^n {}^nC_n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_{n-1} [x^n \text{ का गुणांक}]$$

$$2^n {}^nC_n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_{n-1}$$

$$2^n {}^nC_n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_{n-1}$$

$$2^n {}^nC_n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_{n-1}$$

$$2^n {}^nC_n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_{n-1}$$

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$$2^n {}^nC_n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_{n-1}$$

$$(i) \quad C_0 + 2 \cdot C_1 + 3C_2 + \dots + (n+1)C_n = ?$$

$$(ii) \quad (n+1)2^n$$

$$(iii) \quad (n+1)2^{n-1}$$

$$(iv) \quad (n+2)2^n$$

$$\times C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2 = \begin{cases} 0, \text{ जब } n \text{ विषम हो} \\ (2^{n-1})^2, \text{ जब } n \text{ सम हो} \end{cases}$$

$$\times (1+x)^n = C_0 + C_1 x + C_2 x^2 + \dots + C_n x^n \quad \text{--- (4)}$$

$$\times (1+x)^n = C_0 + C_1 x + C_2 x^2 + \dots + C_n x^n$$

$$\times (1+x)^n = C_0 + C_1 x + C_2 x^2 + \dots + C_n x^n$$

$$\times (1+x)^n = C_0 + C_1 x + C_2 x^2 + \dots + C_n x^n$$

$$\times (1+x)^n = C_0 + C_1 x + C_2 x^2 + \dots + C_n x^n$$

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Q.  $C_0 + 5C_1 + 9C_2 + \dots + 101C_{25} = 2^{25} \times K$  [जब  $k=1$ ]

जहाँ  $(C_0 + 4C_1 + C_2 + \dots + C_{25}) + 4(C_1 + 2C_2 + \dots + 25C_{25}) = 2^{25} \times K$

$2^{25} + 4 \times 25 \times 2^{24} = 2^{25} \times K$

$2^{25} [1 + 25 \times 4] = 2^{25} \times K$

$K = 51$

द्विपद को प्रसार करने। जब हानि विनात्मक या ऋणात्मक हो -  
 - इस स्थिति में किसी द्विपद का प्रसार करने के लिए प्रथम पद को एक व द्वितीय पद को ऋणात्मक रूप से लेनी कम करना होगा।  
 इस स्थिति में प्रसार में पदों की संख्या अन्ततः होती है।

$(x+y)^n$  जब  $x > y = x^n (1 + \frac{y}{x})^n$

$x < y = y^n (1 + \frac{x}{y})^n$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \dots$  (1)

$[|x| < 1, -1 < x < 1]$

$T_{r+1} = \frac{n(n-1)(n-2)\dots(n-r+1)}{r!} x^r$

\*  $(1-x)^n = 1 - nx + \frac{n(n-1)}{2!} x^2 - \frac{n(n-1)(n-2)}{3!} x^3 + \dots$  (2)

$T_{r+1} = \frac{(-1)^r n(n-1)(n-2)\dots(n-r+1)}{r!} x^r$

जहाँ (1) में  $n = -n$  रखने पर

$(1+x)^{-n} = 1 - nx + \frac{n(n+1)}{2!} x^2 - \frac{n(n+1)(n+2)}{3!} x^3 + \dots$  (3)

$T_{r+1} = \frac{(-1)^r n(n+1)(n+2)\dots(n+r-1)}{r!} x^r$

जहाँ (3) में  $x$  के स्थान पर  $(-x)$  रखने पर

$(1-x)^{-n} = 1 + nx + \frac{n(n+1)}{2!} x^2 + \frac{n(n+1)(n+2)}{3!} x^3 + \dots$

$T_{r+1} = \frac{n(n+1)(n+2)\dots(n+r-1)}{r!} x^r$

$(1+x)^{-1} = 1 - x + \frac{1(1+1)}{2!} x^2 - \frac{1(1+1)(1+2)}{3!} x^3 + \dots$

$= 1 - x + \frac{x^2}{2} - \frac{6x^3}{6} + \dots$

$(1-x)^{-2} = 1 + 2x + \frac{2(2+1)}{2!} x^2 + \frac{2(2+1)(2+2)}{3!} x^3 + \dots$

$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots$

$(1-x)^{-1} = 1 + x + \frac{1(1+1)}{2!} x^2 + \frac{1(1+1)(1+2)}{3!} x^3 + \dots$

$= 1 + x + x^2 + x^3 + \dots$

Q.  $(3+2x)$  50 के प्रसार में महत्त्वपूर्ण पद जहाँ  $x = 1/5$

Ans.  $p = \frac{n+1}{1 + \frac{p}{x}} = \frac{51}{1 + \frac{3}{2x}} = \frac{51}{2x + \frac{3}{2}}$

जहाँ  $T_6$  व  $T_7$  महत्त्वपूर्ण पद होगी।

Q.  $(2+3x)^{10}$  के प्रसार में जब  $x = 3/2$  तो तीसरे अंशक पद का पद,

Ans.  $p = \frac{9+1}{1 + \frac{2x}{3x^3}} = \frac{10}{1 + \frac{2}{3x}} = \frac{9 \times 10}{13} = \frac{90}{13} = 6.92 = 6$

जहाँ  $T_6 + 1 = 7$  वाँ पद

$(1+x)^n$  की अवरोध वंश पर दीजिए।

$$P = \frac{n+1}{1+\frac{1}{x}} = n \text{ (अवरोध)}$$

(i) म अवरोध है तो  $T_m$  व  $T_{m+1}$  अवरोधों का अंतर हो अवरोध पर दीजिए।

(ii) म अवरोध नहीं है तो  $T_m + 1$  वंश पर अवरोध दीजिए।

अवरोधों का अंतर हो

Q.30  $(1+x-2x^2)^6 = q_0 + q_1x + q_2x^2 + \dots + q_{12}x^{12}$

$q_2 + q_4 + q_6 + \dots + q_{12} = ?$

$x = 1$  रखते हैं

$0 = q_0 + q_1 + q_2 + q_3 + \dots + q_{12}$

$x = -1$  रखते हैं

$(-1)^6 = q_0 - q_1 + q_2 - q_3 + \dots + q_{12}$

$2^6 = 2q_0 + 2q_2 + 2q_4 + \dots + 2q_{12}$

$2^5 - 1 = q_2 + q_4 + q_6 + \dots + q_{12}$

$31 = q_2 + q_4 + q_6 + \dots + q_{12}$

Q.32  $(1+x+x^2)^n = q_0 + q_1x + q_2x^2 + \dots + q_{2n}x^{2n}$

Put  $x=1$

$3^n = q_0 + q_1 + q_2 + q_3 + \dots + q_{2n}$

$x=w$

$0 = q_0 + q_1w + q_2w^2 + q_3w^3 + \dots + q_{2n}w^{2n}$

$x=w^2$

$0 = q_0 + q_1w^2 + q_2w^4 + q_3w^6 + \dots + q_{2n}w^{4n}$

$3^n = 3q_0 + 3q_2 + 3q_4 + \dots$

$3^{n-1} = q_0 + q_2 + q_4 + \dots$

Q.60

$$\frac{\sqrt{1+x} + 3\sqrt{(1-x)^2} - (1+x)^{1/2} + (1-x)^{3/2}}{1+x + \sqrt{1+x}} = \frac{(1+x)^{1/2} + (1-x)^{3/2}}{1+x + (1+x)^{1/2}}$$

$$= \frac{1 + \frac{1}{2}x + 1 - \frac{2}{3}x}{1+x + 1 + \frac{1}{2}x} = \frac{2 - \frac{x}{3}}{2 + \frac{3}{2}x} = \left(\frac{1-x}{12}\right) \left(\frac{1+3}{4}\right)^{1/2}$$

$$= \left(\frac{1-x}{12}\right) \left(\frac{1-\frac{3}{4}x}{4}\right) = \frac{1-3x-x+\frac{3}{4}x^2}{12} = \frac{1-3x-x+\frac{3}{4}x^2}{12}$$

$$= 1 - \frac{5}{6}x \text{ पर } x^2 = \frac{3x}{4} \text{ according to Q.60}$$

$$\left(\frac{1+3x}{4}\right)^{-4} \int \sqrt{16-3x} = \frac{1}{(8+x)^{2/3}} = P+Qx$$

$$\left(\frac{1+\frac{3}{4}x}{4}\right)^{-4} \left(16-3x\right)^{1/2} = (1-3x)^{-4} \left(1-\frac{3}{32}x\right) (8+x)^{2/3}$$

$$(8+x)^{2/3} = (1+x)^{2/3}$$

$$\frac{1 - \frac{3}{32}x - 3x}{(8+x)^{2/3}} = \frac{(1-\frac{91}{32}x)}{(8+x)^{2/3}} \left(\frac{1-\frac{3}{32}x}{(8+x)^{2/3}}\right)$$

$$\frac{1+x}{12} = \frac{1-x-\frac{59x}{32}}{12} = \frac{1-x-\frac{59x}{32}}{12}$$

$$1 - \frac{66x-297x}{96} = \frac{1-305x}{96}$$

$$P+Qx = \frac{1-305x}{96}$$

$$P=1 \quad Q = -\frac{305}{96}$$



(43)

अब d = ...

$$a(a-b) + (a+b)(a-b) + \dots + (n+1)(n) - d [C_1 - 2C_2 + 3C_3 - 4C_4 + \dots]$$

WKT  $C_0 - C_1 + C_2 - C_3 + \dots = 0$

$$(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + \dots$$

अब हमें इसका उपयोग करें

$$n(1+x)^{n-1} = C_1 + 2C_2x + 3C_3x^2 + \dots$$

$$x = -1$$

$$0 = nC_1 - 2nC_2 + 3nC_3 - \dots$$

$$0 [0] - d [0] = 0 \text{ Ans } [n \neq 1]$$

(44)  $(1+x)^{15} = C_0 + C_1x + C_2x^2 + C_3x^3 + C_4x^4 + \dots + C_{15}x^{15}$

अब हमें इसका उपयोग करें

$$15(1+x)^{14} = C_1 + 2C_2x + 3C_3x^2 + 4C_4x^3 + \dots$$

$$x = 1$$

$$15 \cdot 2^{14} = C_1 + 2C_2 + 3C_3 + 4C_4 + \dots$$

$$2^{14} = C_1 + C_1 + C_2 + C_2 + C_3 + C_3 + \dots$$

$$15 \cdot 2^{14} - 2^{14} = 14C_1 + 14C_2 + 14C_3 + \dots$$

$$15 \cdot 2^{14} - 2^{14} = C_2 + 3C_3 + 4C_4 + \dots$$

$$2^{14} \times 15 + 1 = C_2 + 3C_3 + 4C_4 + \dots$$

\* अब हमें इसका उपयोग करें :-

$$(x+y+z)^n \text{ का } n\text{I.T. } T_{r+1} = n C_r x^r y^r z^r$$

$$= \frac{n!}{r!r!r!} x^r y^r z^r$$

$$= \frac{n!}{r!r!r!} x^{r_1} y^{r_2} z^{r_3} \text{ जहाँ } r_1 + r_2 + r_3 = n$$

$$0 \leq r_1, r_2 \leq n \text{ एवं } r_1, r_2 \in \mathbb{N}$$

$$(x+y+z)^n \text{ का } n\text{I.T. } T_{r+1} = \frac{n!}{r_1!r_2!r_3!} x^{r_1} y^{r_2} z^{r_3}$$

$$\text{जहाँ } r_1 + r_2 + r_3 = n, 0 \leq (r_1, r_2, r_3) \leq n$$

$$(x_1 + x_2 + x_3 + \dots + x_n)^n \text{ का } n\text{I.T.}$$

$$T_{r+1} = \frac{n!}{r_1!r_2!r_3! \dots r_m!} x_1^{r_1} x_2^{r_2} x_3^{r_3} \dots x_m^{r_m}$$

$$\text{जहाँ } r_1 + r_2 + r_3 + \dots + r_m = n$$

$$0 \leq r_1, r_2, r_3, \dots, r_m \leq n$$

Q  $(x+y+z)^9$  के अक्षरों में  $x^2y^3z^4$  का गुणांक ज्ञात करें

$$\frac{9!}{2!3!4!}$$

Q  $(a+bx+cx^2)^8$  के अक्षरों में  $a^5b^4c^2$  का गुणांक ज्ञात करें

$$\frac{8!}{5!2!1!} \cdot \frac{8!}{2!3!3!} \cdot \frac{8!}{2!3!3!}$$

$$\frac{8!}{5!2!1!} = 8 \quad 2a + 3b + 4c = 8 \quad r_1 + r_2 = 7$$

$$r_1 = 3, r_2 = 4, r_3 = 1$$

$$\frac{8!}{3!4!1!}$$