Now, recall the basic "rules" for the form of the series answer. We don't want anything out in front of the series and we want a single *x* with a single exponent on it.

These are easy enough rules to take care of. All we need to do is move whatever is in front of the series to the inside of the series and use basic exponent rules to take care of the *x* "rule". Doing all this gives,

$$x^6 \mathbf{e}^{2x^3} = x^6 \sum_{n=0}^{\infty} \frac{\left(2x^3\right)^n}{n!} = \sum_{n=0}^{\infty} x^6 \frac{2^n \left(x^3\right)^n}{n!} = \boxed{\sum_{n=0}^{\infty} \frac{2^n x^{3n+6}}{n!}}$$

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3. Find the Taylor Series for $f\left(x\right) = \mathbf{e}^{-6x}$ about $x = -4$.

Step 1
Because we are working about $x = -4$ in this problem we are not able to just use the formula derived in class for the exponential function because that requires us to be working about $x = 0$.

Step 2
So, we'll need to start over from the beginning and start taking some derivatives of the function.

$$\begin{aligned}
n = 0: &\quad f\left(x\right) = \mathbf{e}^{-6x} \\
n = 1: &\quad f'\left(x\right) = -6\mathbf{e}^{-6x} \\
n = 2: &\quad f''\left(x\right) = \left(-6\right)^2 \mathbf{e}^{-6x} \\
n = 3: &\quad f^{(3)}\left(x\right) = \left(-6\right)^3 \mathbf{e}^{-6x} \\
n = 4: &\quad f^{(4)}\left(x\right) = \left(-6\right)^4 \mathbf{e}^{-6x}
\end{aligned}$$

Remember that, in general, we're going to need to go out to at least $n = 4$ for most of these problems to make sure that we can get the formula for the general term in the Taylor Series.

Also, remember to NOT multiply things out when taking derivatives! Doing that will make your life much harder when it comes time to find the general formula.

Step 3
It is now time to see if we can get a formula for the general term in the Taylor Series.

In this case, it is (hopefully) pretty simple to catch the pattern in the derivatives above. The general term is given by,

$$f^{(n)}\left(x\right) = \left(-6\right)^n \mathbf{e}^{-6x} \qquad n = 0,1,2,3,\ldots$$

As noted this formula works all the way back to $n = 0$. It is important to make sure that you check this formula to determine just how far back it will work. We will, on occasion, get formulas that will not work for the first couple of *n*'s and we need to know that before we start writing down the Taylor Series.

Step 4
Now, recall that we don't really want the general term at any *x*. We want the general term at $x = -4$. This is,

$$f^{(n)}\left(-4\right) = \left(-6\right)^n \mathbf{e}^{24} \qquad n = 0,1,2,3,\ldots$$

Step 5
Okay, at this point we can formally write down the Taylor Series for this problem.

$$\mathbf{e}^{-6x} = \sum_{n=0}^{\infty} \frac{f^{(n)}\left(-4\right)}{n!}\left(x+4\right)^n = \boxed{\sum_{n=0}^{\infty} \frac{\left(-6\right)^n \mathbf{e}^{24}}{n!}\left(x+4\right)^n}$$

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4. Find the Taylor Series for $f\left(x\right) = \ln\left(3+4x\right)$ about $x = 0$.

Step 1
Okay, we'll need to start off this problem by taking a few derivatives of the function.

$$\begin{aligned}
n=0: &\quad f\left(x\right) = \ln\left(3+4x\right)\\
n=1: &\quad f'\left(x\right) = \frac{4}{3+4x} = 4\left(3+4x\right)^{-1}\\
n=2: &\quad f''\left(x\right) = -4^2\left(3+4x\right)^{-2}\\
n=3: &\quad f^{(3)}\left(x\right) = 4^3\left(2\right)\left(3+4x\right)^{-3}\\
n=4: &\quad f^{(4)}\left(x\right) = -4^4\left(2\right)\left(3\right)\left(3+4x\right)^{-4}\\
n=5: &\quad f^{(5)}\left(x\right) = 4^5\left(2\right)\left(3\right)\left(4\right)\left(3+4x\right)^{-5}
\end{aligned}$$

Remember that, in general, we're going to need to go out to at least $n = 4$ for most of these problems to make sure that we can get the formula for the general term in the Taylor Series.

Also, remember to NOT multiply things out when taking derivatives! Doing that will make your life much harder when it comes time to find the general formula. In this case we "merged" all the 4's that came from the chain rule into a single term but left it as an exponent rather than get an actual value. This is not uncommon with these kinds of problems. The exponents we dropped down for the derivatives we left alone with the exception of dealing with the signs.

Step 2