Section 4-16 : Taylor Series

1. Use one of the Taylor Series derived in the notes to determine the Taylor Series for $f\left(x\right)=\cos\left(4x\right)$ about $x=0$.

Step 1
There really isn't all that much to do here for this problem. We are working with cosine and want the Taylor series about $x=0$ and so we can use the Taylor series for cosine derived in the notes to get,

$$\cos\left(4x\right)=\sum_{n=0}^{\infty}\frac{\left(-1\right)^{n}\left(4x\right)^{2n}}{\left(2n\right)!}$$

Step 2
Now, recall the basic "rules" for the form of the series answer. We don't want anything out in front of the series and we want a single *x* with a single exponent on it.

In this case we don't have anything out in front of the series to worry about so all we need to do is use the basic exponent rules on the 4*x* term to get,

$$\cos\left(4x\right)=\sum_{n=0}^{\infty}\frac{\left(-1\right)^{n}4^{2n}x^{2n}}{\left(2n\right)!}=\boxed{\sum_{n=0}^{\infty}\frac{\left(-1\right)^{n}16^{n}x^{2n}}{\left(2n\right)!}}$$

---

2. Use one of the Taylor Series derived in the notes to determine the Taylor Series for $f\left(x\right)=x^{6}\mathbf{e}^{2x^{3}}$ about $x=0$.

Step 1
There really isn't all that much to do here for this problem. We are working with the exponential function and want the Taylor series about $x=0$ and so we can use the Taylor series for the exponential function derived in the notes to get,

$$x^{6}\mathbf{e}^{2x^{3}}=x^{6}\sum_{n=0}^{\infty}\frac{\left(2x^{3}\right)^{n}}{n!}$$

Note that we only convert the exponential using the Taylor series derived in the notes and, at this point, we just leave the x^{6} alone in front of the series.

Step 2

Now, recall the basic "rules" for the form of the series answer. We don't want anything out in front of the series and we want a single *x* with a single exponent on it.

These are easy enough rules to take care of. All we need to do is move whatever is in front of the series to the inside of the series and use basic exponent rules to take care of the *x* "rule". Doing all this gives,

$$x^6\mathbf{e}^{2x^3} = x^6\sum_{n=0}^{\infty}\frac{\left(2x^3\right)^n}{n!} = \sum_{n=0}^{\infty}x^6\,\frac{2^n\left(x^3\right)^n}{n!} = \boxed{\sum_{n=0}^{\infty}\frac{2^n\,x^{3n+6}}{n!}}$$