6. Find the Taylor Series for $f(x)=7x^2-6x+1$ about $x=2$.

Step 1

First, let's not get too excited about the fact that we have a polynomial here for this problem. It works exactly the same way with a few small differences.

We'll start off by taking a few derivatives of the function and evaluating them at $x=2$

$$\begin{array}{lll} n=0: & f(x)=7x^2-6x+1 & f(2)=17 \\ n=1: & f'(x)=14x-6 & f'(2)=22 \\ n=2: & f''(x)=14 & f''(2)=14 \\ n\geq 3: & f^{(n)}(x)=0 & f^{(n)}(2)=0 \end{array}$$

Okay, this is where one of the differences between a polynomial and the other types of functions we typically see with Taylor Series problems. After some point all the derivatives will be zero. That is not something to get excited about. In fact, it actually makes the problem a little easier!

Because all the derivatives are zero after some point we don't need a formula for the general term. All we need are the values of the non-zero derivative terms.

Step 2

Once we have the values from the previous step all we need to do is write down the Taylor Series. To do that all we need to do is strip all the non-zero terms from the series and then acknowledge that the remainder will just be zero (all the remaining terms are zero after all!).

Doing this gives,

$$\begin{aligned} 7x^2-6x+1 &= \sum_{n=0}^{\infty}\frac{f^{(n)}(2)}{n!}(x-2)^n \\ &= f(2)+f'(2)(x-2)+\tfrac{1}{2}f''(2)(x-2)^2+\sum_{n=3}^{\infty}\frac{f^{(n)}(2)}{n!}(x-2)^n \\ &= \boxed{17+22(x-2)+7(x-2)^2} \end{aligned}$$

It looks a little strange but there it is. Do not multiply/simplify this out. This really is the answer we are looking for.

Also, don't think that this is a problem that is just done to make you work another problem. There are applications of series (beyond the scope of this course however…) that really do require this kind of thing to be done as strange as that might sound!