5. The blades of a windmill sweep out a circle of area A.

a. If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through it in time t?
b. What is the kinetic energy of the air?
c. Assume that the windmill converts 25% of the wind's energy into electrical energy, and that A = 30 m^2, v = 36 km/h and the density of air is 1.2 kg m^{-3}. What is the electrical power produced?

Sol. According to the question Area of the circle swept by the windmill = A
Velocity of the wind = v
Density of air = ρ
a. Volume of the wind flowing through the windmill per sec = Av
Mass of the wind flowing through the windmill per sec = ρAv
Mass m, of the wind flowing through the windmill in time t = ρAvt
b. Kinetic energy of air $= \frac{1}{2}mv^2$
$KE = \frac{1}{2}(\rho Avt)v^2 = \frac{1}{2}\rho Av^3t$
According to the question , area of the circle swept by the windmill , A = 30 m^2
Velocity of the wind = v= 36 km/h = 10 m/s
Density of air, ρ = 1.2 kg m^{-3}
As per the question, $E_{electric} = E_{wind}$
$E_{electric} = \frac{25}{100} \times \frac{1}{2}\rho Av^3t = \frac{1}{8}\rho Av^3t$
Electrical power, $P_{electric} = \frac{E_{electric}}{t}$
$P_{electric} = \frac{1}{8}\frac{\rho Av^3t}{t} = \frac{1}{8}\rho Av^3$
$P_{electric} = \frac{1}{8} \times 1.2 \times 30 \times (10)^3$
$P_{electric} = 4.5 \times 10^3 W = 4.5kW$